

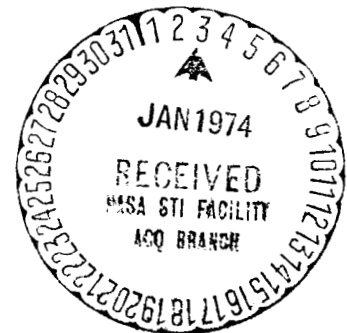
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Briefing Notes on Presentation
to NASA at Ames Research Center
on the Multi-Attribute Decision
Problem

by

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I. INTRODUCTION

- MULTI-ATTRIBUTE DECISION PROBLEMS
- EXAMPLES OF ATTRIBUTES IN SPACE MISSION PLANNING
- DECISION MAKER'S PREFERENCE STRUCTURE

Introduction

The topic of my research is the multi-attribute decision problem. This problem arises when the objectives and alternative solutions designed to satisfy the objectives are such that they give rise to several distinct types of outcomes. Furthermore, it is assumed that no reasonable model can be postulated which relates these outcomes to one common numeraire (for example, dollars) which can be employed to measure the total level of output of each alternative. The problem is to determine the trade-offs between these outcomes, or attributes, so that one alternative can be selected. A complicating factor is that the trade-offs may not be constant over the portion of the outcome vector space of interest. That is, at some point specified by the level of output of each attribute (i.e., a vector in the outcome space) there exist trade-offs which can be used to derive new points in the neighborhood such that all points in this neighborhood are equally preferred to each other. However, if we move to some point not in the neighborhood of the old point, the same trade-off ratios may no longer be valid, and a new set of trade-off ratios must be derived.

One assumption necessary to the solution of this type of problem is that there exists, if only internally to the decision-maker, a preference structure on all points in the outcome space. That is, if we choose any two points, either one is preferred to the other, or they are equally preferred, or the preference is for the second point over the first. There are a number of axioms which we can reasonably expect the preferences to obey. These will not be explicitly enumerated here, but some of the more important ones will be mentioned later when the multi-attribute problem is graphically defined. The important point is that if the preference structure obeys these axioms (i.e., is fairly well-behaved) there exists a real valued function which can be used to encode the preferences. The greater the value of this function, the more desirable the outcome vectors which generate it. If this function were explicitly known, the multi-attribute problem would be a fairly straightforward optimization problem which would require very little of the decision-maker's time. Unfortunately, in many decision problems the preference function is only known implicitly by the decision-maker. Note that the preference function I am discussing is not necessarily the preference function of the

collective benefactors of the project. However, it is this preference function as perceived by the decision-maker who may or may not be one of the benefactors.

The multi-attribute problem will probably be more and more encountered in space mission planning as objectives other than overriding ones, such as landing a man on the moon, become important. Examples of outcome variables for space mission planning may include the following:

- 1) Number of man-hours in space.
- 2) Number of scientist-hours in space.
- 3) Quantities of various types of scientific data obtained.
- 4) Quality of various types of scientific data obtained.
- 5) Quantities of various types of earth resources survey data obtained.
- 6) Duration of man-in-space missions.

Obviously, the list can go on and on. However, such a list can be used as an aid in defining the important outcome variables for the specific set of alternatives being considered. Note that these outcome variables should be quantifiable items so that trade-off ratios can be defined. The selection of the proper set of outcome variables is a critical part of the problem solution which is not dealt with in this presentation.

II. OUTLINE OF PRESENTATION

- ITERATIVE APPROACH AND SPECIAL PROBLEMS
- THE ITERATIVE METHOD
- COORDINATE DESCENT METHOD
- THE SECANT METHOD
- DISCRETE ALTERNATIVE SETS

Outline of Presentation

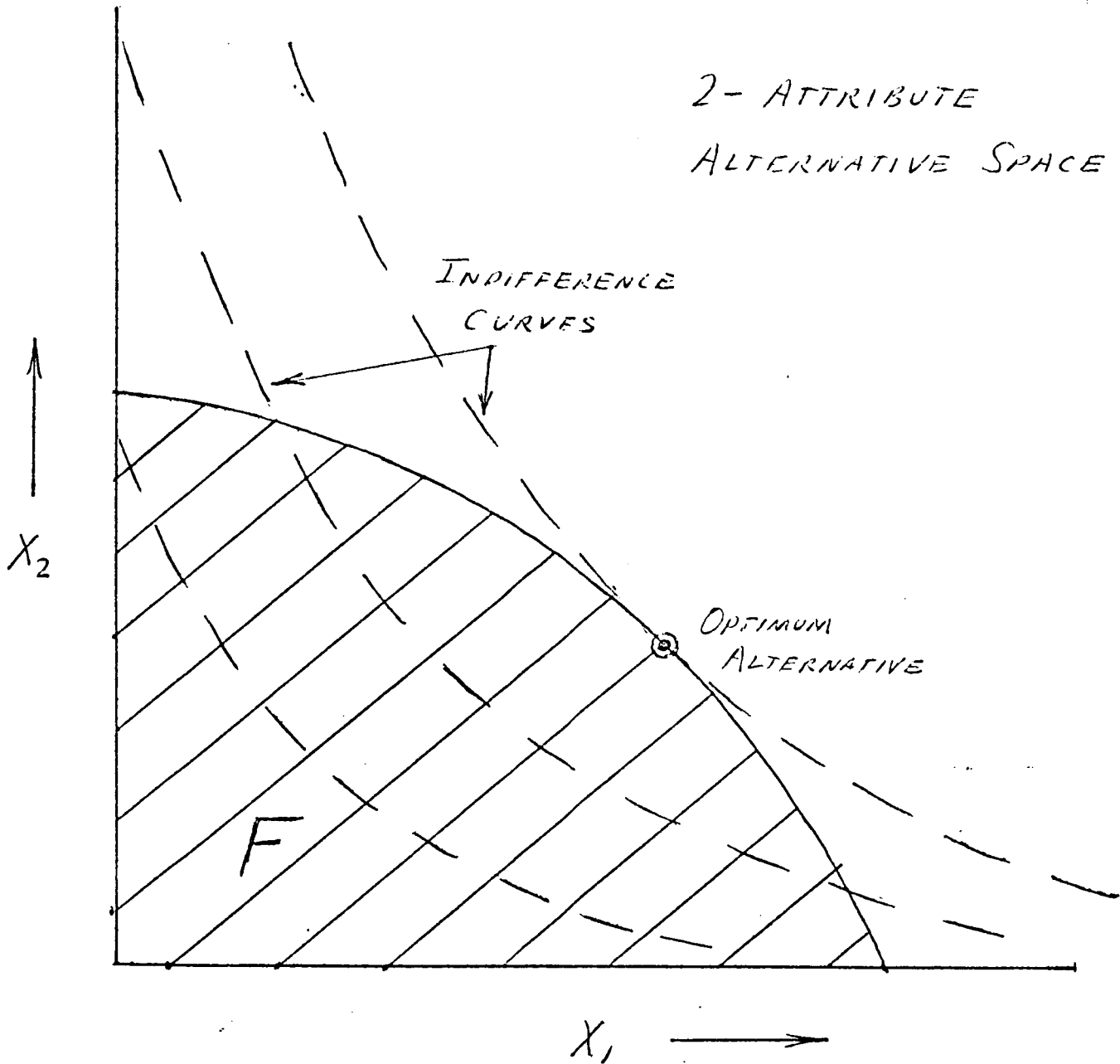
The multi-attribute problem suggests two basic approaches to its solution. One approach would be to interview the decision-maker to obtain information about his preferences so that a preference function can be derived. As the number of attributes and the set of alternatives increase, the amount of information required to construct the preference function and the amount of decision-maker's time required increases very rapidly and may become infeasible. Once the preference function is available, non-linear optimization techniques can then be applied to obtain the best alternative.

A second approach is an iterative approach. In this approach we attempt to converge toward the solution as we obtain information from the decision-maker. When we are close enough to the solution we can decide to stop and not require any further preference information. Thus, the amount of information required is related to the rate at which we converge to the solution, and in a sense we minimize the amount of information required. In contrast, the first approach requires us to obtain enough information to adequately construct what may be a fairly complex preference function over a significant portion of the outcome space.

The second approach is the subject of this presentation. The potential benefits of this approach compared to the preference function construction approach will become clearer as we get further into the presentation.

The outline of the remainder of the presentation is the following. First, I will define and discuss the general iterative approach and some special problems that may arise and how to handle these. Next, I will discuss and present examples of three iterative algorithms called the iterative method, the coordinate descent method, and the secant method. Results on sample problems using these three methods will be presented and compared. Although the presentation up to this point will be concerned with continuous alternative sets, I will conclude the presentation by indicating how these methods can be applied to discrete alternative sets.

2-ATTRIBUTE
ALTERNATIVE SPACE



III. 2-ATTRIBUTE DECISION PROBLEM

Two-Attribute Decision Problem

This figure serves to define the multi-attribute problem in terms of a two-dimensional example. We label the two outcome variables x_1 and x_2 respectively, and represent the outcome space of interest as the positive quadrant of the cartesian space. This implies the assumption that the two outcome variables are such that only positive output quantities are considered. In general, it is conceivable that negative quantities of certain types of outcome variables may be considered. However, we can always redefine the origin of our coordinates so that the points of interest are again in the positive quadrant. Another important assumption is that the outcome variables are so defined that they are desirable outcomes in the sense that more of any one outcome variable is preferred to less. In some cases where negative outcomes are desirable, it may be necessary to redefine an outcome variable as the negative of the previous outcome variable.

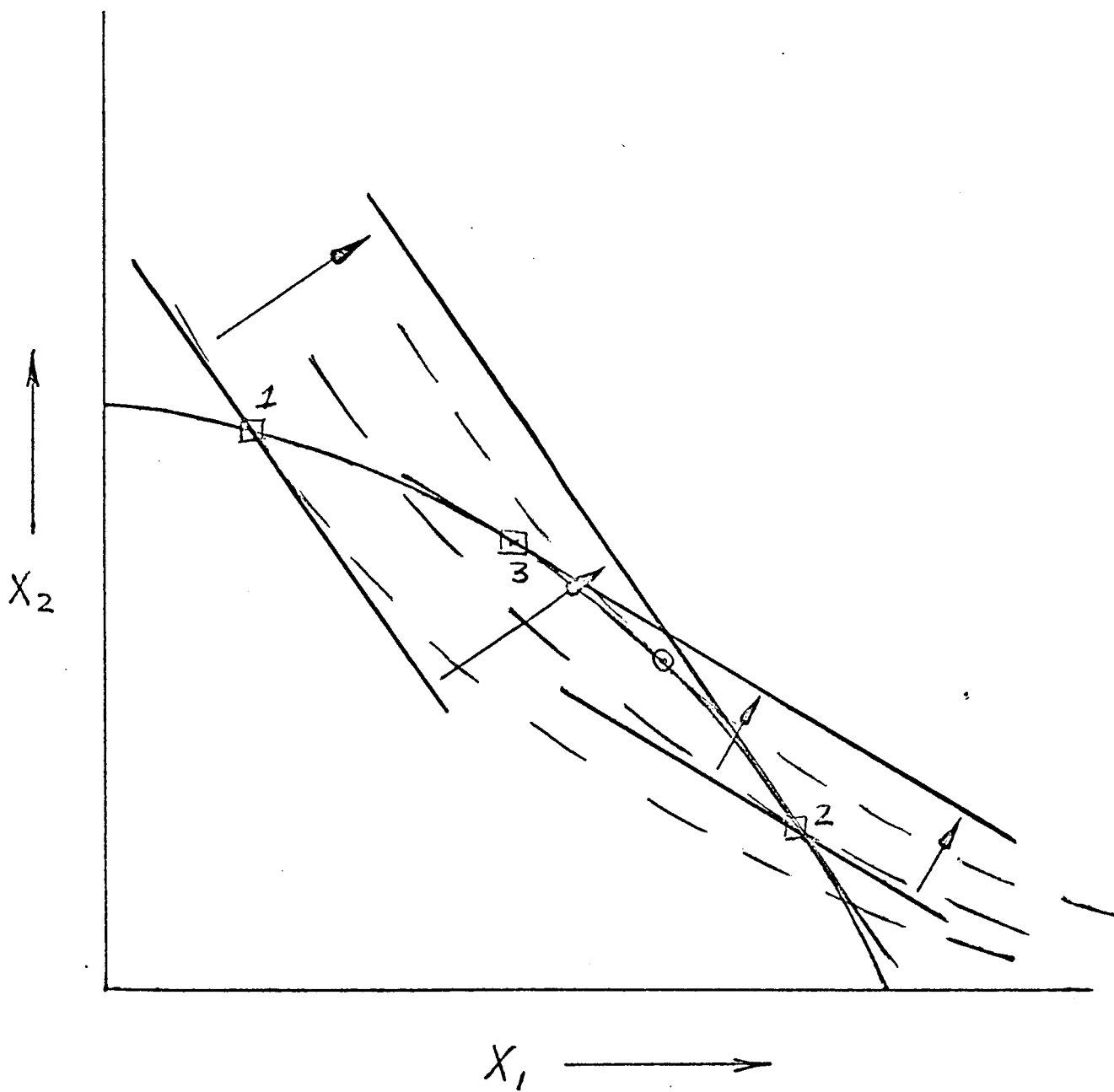
The cross-hatched convex set labelled F represents the set of outcome vectors of all feasible alternatives. Note that only the upper right boundary of this set needs to be considered since under the assumption that more is preferred to less, we must find our solution on this boundary. I will interchangeably refer to the set F or its boundary as the feasible set.

The dashed curves in this figure represent the decision-maker's preferences and are called indifference curves. An indifference curve defines a set of outcome vectors such that any point in the set is neither more nor less preferred to any other point in the set. There is actually an infinite family of such indifference curves that cover the entire outcome space. The indifference curves can be considered as contour lines of a hill whose base lies in the plane of the figure and which rises normal to this plane. Under this interpretation, higher contour lines are preferred to lower contour lines, and we desire to find the point in the feasible set which lies on the highest contour. If the elevation of each contour increases as we proceed toward the upper right of the figure, the optimum point is that labeled as optimum in the figure.

The assumptions that the indifference curves are convex as shown, and show increasing preferences toward the upper right direction are important

but reasonable assumptions in the iterative approaches to be discussed. These features are obtained if the outcome variables are defined as desirable quantities (in the sense defined above) and if the decision-maker's preferences are such that as less of one outcome variable is available, a further decrease in the quantity of that variable requires more of an increase in the other variable for him to remain indifferent than if more of the outcome variable were initially available.

The figure shows that the condition for an optimum is that at the optimum, the slope of the line tangent to the feasible set must be equal to the slope of the line tangent to the indifference curve. In an n -th dimensional problem there are $(n-1)$ such slopes for the feasible set and for the indifference curve. Thus, the condition for an optimum is that all $(n-1)$ pairs of slopes be simultaneously equal.



IV. SOLUTION BY ITERATION

Solution by Iteration

This figure serves to define the basic iterative approach, and shows an example of how it would work. Before I discuss this aspect, I would like to point out that the non-iterative approach would involve obtaining enough information from the decision-maker to construct good approximations to any of the infinite family of indifference curves which cover the outcome variable space. One can easily imagine the order of magnitude of this task as the dimensionality of the problem increases.

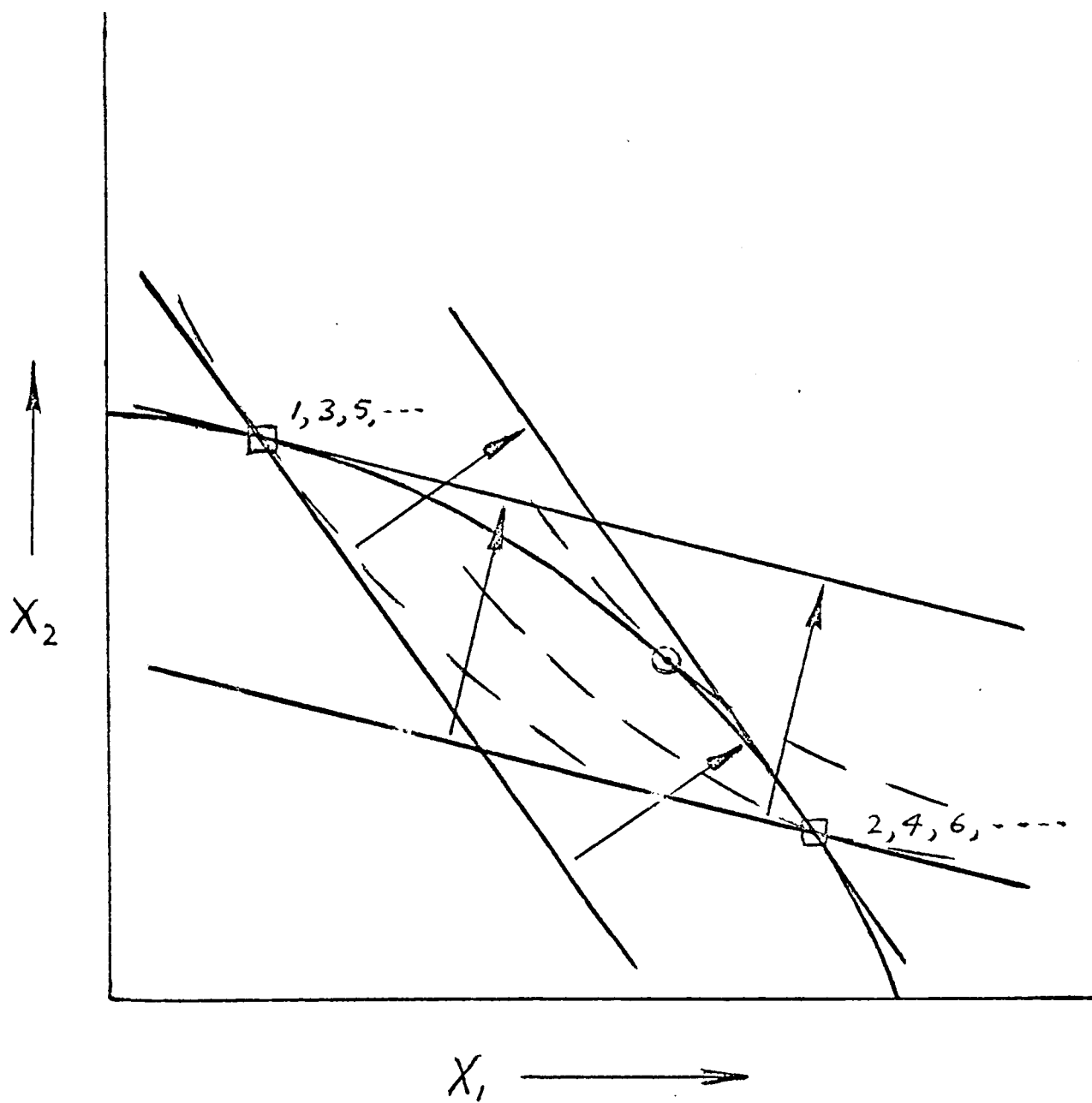
To illustrate the iterative approach, this figure shows the feasible set (the solid curve), three of the indifference curves (the three dashed curves), and the solution point, which is circled. Note that the indifference curves are not really available to the decision analyst.

The iterative approach proceeds as follows. The analyst selects a point, labeled 1, which based on all his prior information is the best candidate for the solution. The analyst then interviews the decision-maker to determine the decision-maker's trade-off ratio between the outcome variables x_1 and x_2 . One way to obtain this trade-off ratio is to ask the decision-maker to imagine that the outcome vector represented by point 1 has been obtained by choosing the corresponding alternative. Next, present the decision-maker with an arbitrary incremental decrease in x_1 , and ask him how much x_2 would have to be increased in order to offset the loss in x_1 . Then determine whether a smaller decrease in x_1 would decrease the increase in x_2 required by the same proportion. If not, the incremental decrease in x_1 must be decreased further. If the proportion does remain the same, the ratio of the increase in x_2 to the decrease in x_1 defines the trade-off ratio. Note that this trade-off ratio is the slope of the tangent line to the indifference curve at point 1. This tangent line through point 1 is shown in the figure. If the slope of the tangent line to the feasible set (not shown in the figure) equals this trade-off ratio, then we are at the solution and we stop, and no further information is required about the preferences of the decision-maker. Since in this example the slopes are not equal, we know that we are not at the solution and we try to determine a new point which is closer to the solution (one measure of the closeness to the solution is the magnitude of the difference between the two slopes).

The basic iterative approach is to assume that the tangent line representing the trade-off ratio is a good approximation to the indifference curve, and further that the slope of the indifference curves remains constant over the outcome space of interest. This assumption allows us to define a sub-optimization problem which can be numerically solved to yield the point labeled 2. Graphically the solution is obtained by sliding the tangent line to the indifference curve in the direction of the arrows to a point where it becomes a tangent line to the feasible set. This condition is met at point 2.

We now repeat the iteration cycle by obtaining the decision-maker's trade-off ratios at point 2, determining whether the solution condition is met, and if not, solving a new sub-optimization problem. In the example shown, point 2 is not a solution, and point 3 is the next point we consider. We see that we are converging to the solution, and in addition we see that each subsequent point is strictly preferred to the previous point. This last bit of knowledge is not generally available to the analyst but is one important property to insure convergence.

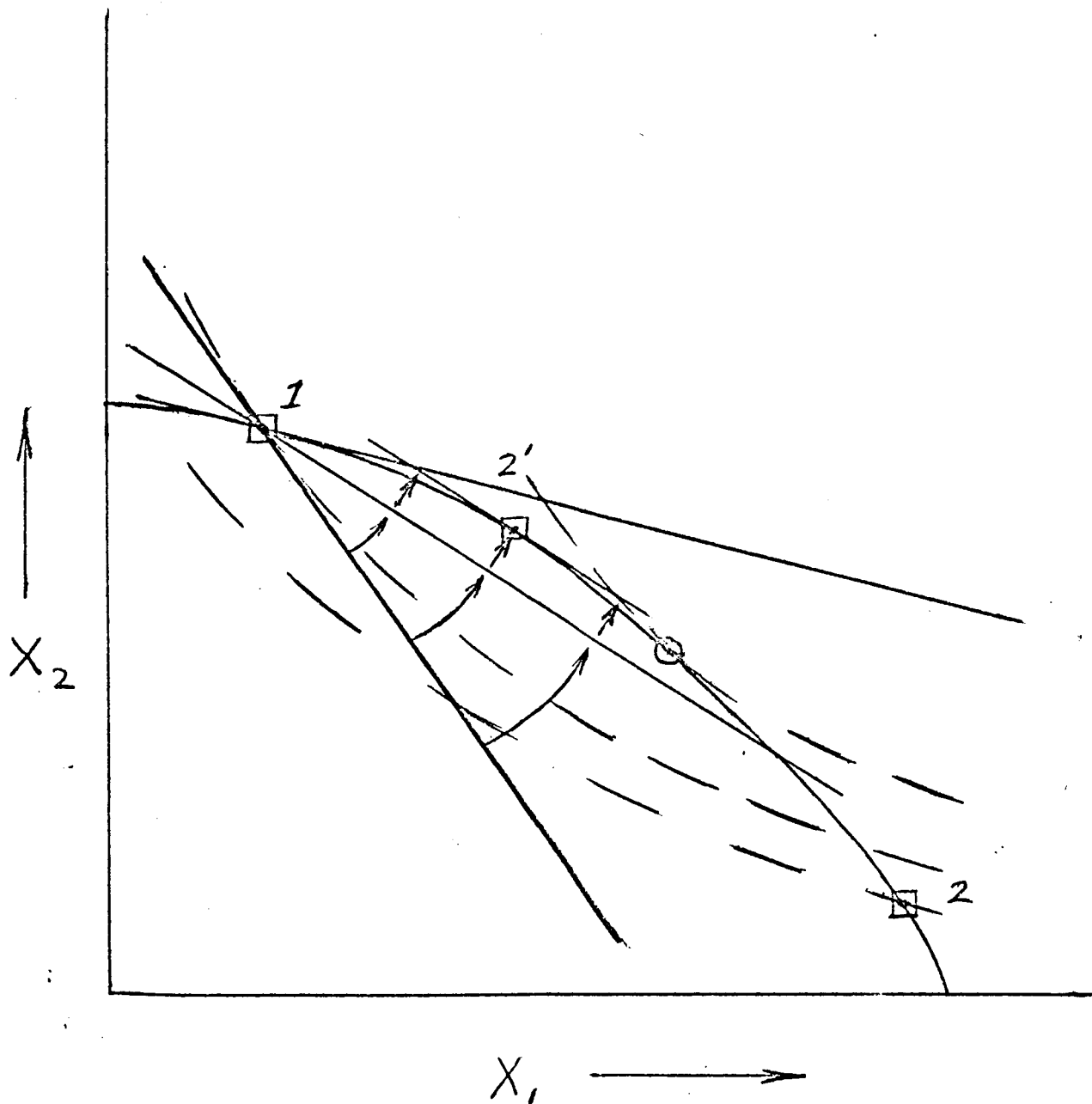
Note that if our prior information is such that a point closer to the solution is initially chosen, very few iterations may be required, implying very little required information about the preference structure.



V. OSCILLATION

Oscillation

One of the special problems which arises with this approach is called oscillation. This occurs when as shown in the figure the slope of the indifference curve at one point equals the slope of the feasible set at a second point, and vice versa. The iteration cycle previously described will then oscillate between the two points as indicated.



VI. ITERATION WITH
RELAXATION

Iteration with Relaxation

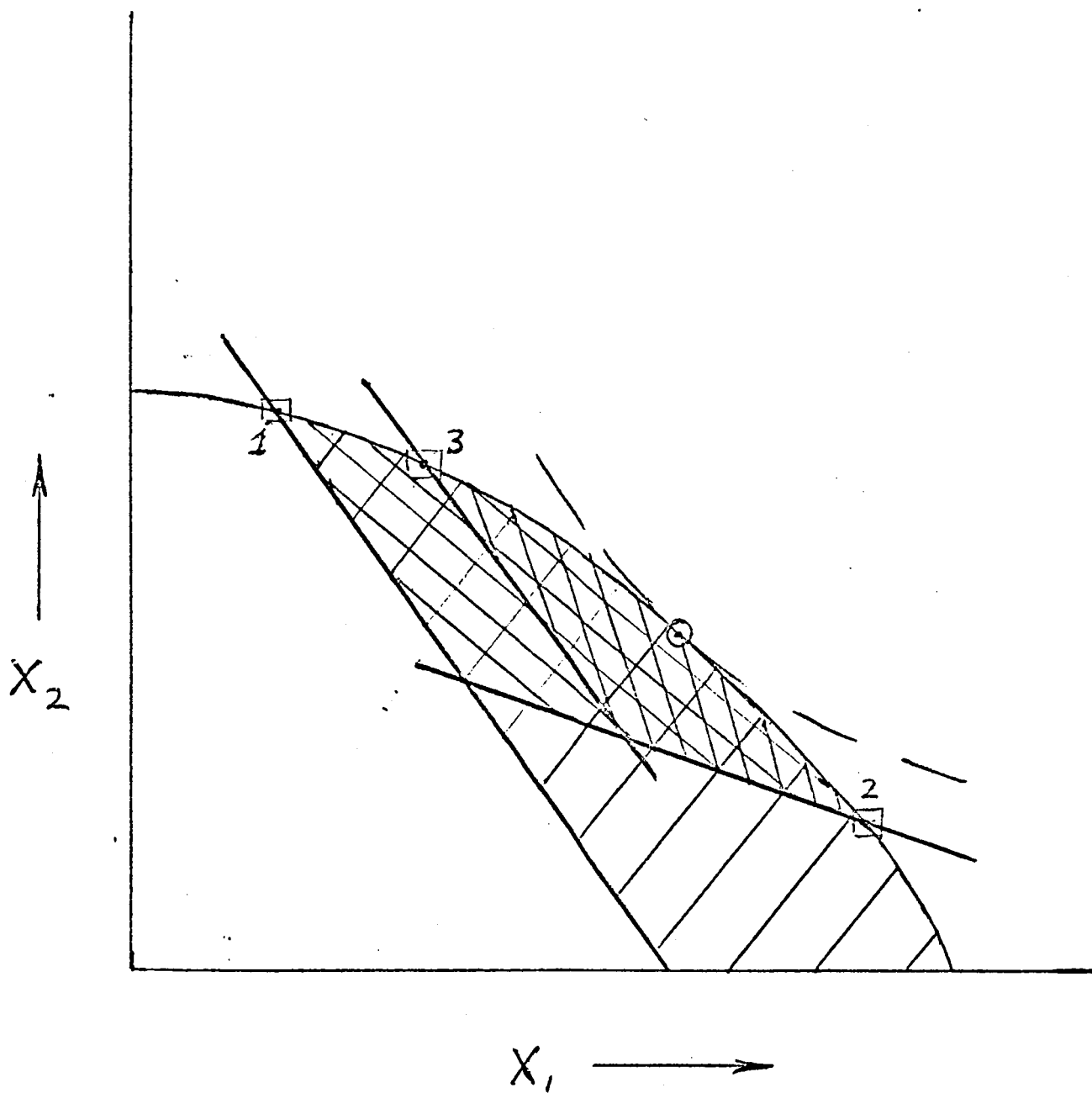
This figure shows a case where divergence may occur and a method for assuring convergence. In the case shown, we start at point 1 and obtain the trade-off ratio represented by the solid line through point 1 tangent to the indifference curve through point 1. Using the iteration previously defined the next point would be point 2. However, note that point 2 lies on an indifference curve which is closer to the origin than the indifference curve through point 1. This means that point 2 is less preferred than point 1. Thus, it is not hard to imagine a case where we continually obtain less preferred points and must diverge from the solution. To insure convergence we would like each successive point to be strictly preferred to the previous point.

One approach to guarantee obtaining a point preferred to point 1 can be derived as follows. Notice that at point 1 we have two tangent lines which are both shown in the figure. One is tangent to the indifference curve and the other is tangent to the feasible set. Any slope between these two slopes can be used in the sub-optimization problem, and as we continuously vary the slope between these limits we obtain solutions to the sub-optimization problem which lie between point 1 and point 2 on the feasible set. The third solid straight line through point 1 is one such slope intermediate between the limiting slopes. It can be thought of as a rotation of the trade-off ratio slope as indicated by the arrows. Using this slope to define the sub-optimization problem results in the point 2'. In this case we can see that point 2' is strictly preferred to point 1.

It can be shown that there exists some slope intermediate between the two limiting slopes such that the solution to the corresponding sub-optimization problem will be strictly preferred to point 1. Thus, we are assured that if point 1 is not the solution, a more preferred point can be obtained for the next iteration. The question is that if we obtain a point such as point 2', how do we know that it is preferred to point 1? At this point the answer would appear to be that we must ask the decision-maker his preference between the two points. If a point generated in this manner were not preferred to the generating point, then the intermediate slope would have to be further rotated, or relaxed, until a preferred point is obtained. The

intermediate slope is obtained by taking a linear combination of the two limiting slopes such that the two weighting coefficients sum to one. Thus, for the two-dimensional problem either weighting coefficient specifies the new slope. The coefficient which is the weighting factor for the trade-off ratio is called the relaxation coefficient. Thus, a relaxation coefficient of 1.0 specifies the trade-off ratio, while a relaxation coefficient of 0.0 specifies the slope of the line tangent to the feasible set.

This approach in its present form is not very satisfactory on two counts. First, the decision-maker must be asked to state his preference between two vectors for each point. This may be difficult and time-consuming for the decision-maker. Indeed it will be at least as difficult as specifying the trade-off ratios at a given point, and perhaps not really feasible. Second, since a relaxation coefficient must be determined by a trial-and-error type of search procedure, the decision-maker may be forced to make many preference assessments between pairs of vectors before the next iteration point is obtained.



VII. ALTERNATE DESCENT
FUNCTION

Alternate Descent Function

The previous discussion revolved around obtaining a descent function for assuring convergence. The descent function is a function of each iteration point in the outcome, whose minimization is equivalent to solving the underlying optimization problem. New iteration points are accepted only if the value of the descent function decreases from the previous value. In the previous discussion, the descent function was assumed to be the negative of the preference function.

This figure illustrates an alternate descent function which can be employed. Note that only one indifference curve is shown to indicate the optimum point. The three solid line segments represent tangent lines (i.e., trade-off ratios) to the indifference curves at each of the three points shown. Consider beginning the iterations at point 1. The trade-off ratio line through point 1 can be alternately thought of as a hyperplane which divides the entire outcome space into two halves, that above the hyperplane (i.e., to the upper right) and that below the hyperplane. Due to the convexity assumption on the indifference curves, and the direction of increasing preferences, we know that all points strictly preferred to point 1 must lie above the hyperplane (to be sure, all equally preferred points and some less preferred points also lie on or above the hyperplane, but all points below are definitely less preferred). Our solution must therefore lie on the portion of the feasible alternative set above the hyperplane. A convenient measure of this subset is the union of the singly, doubly, and triply cross-hatched areas shown.

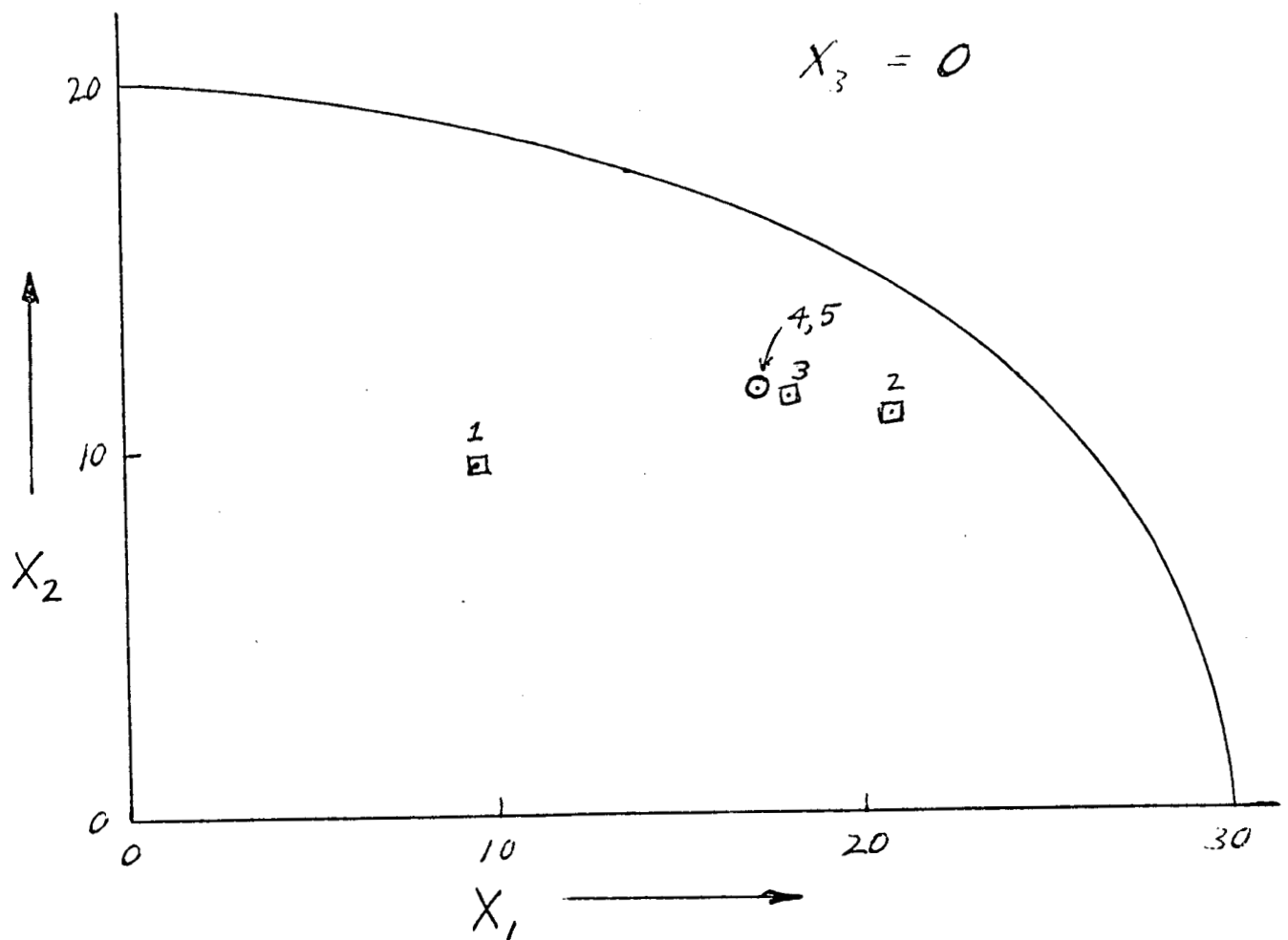
Consider now any point, say point 2, which lies above the hyperplane at point 1. If point 2 is not the solution, then by a similar argument as above, we obtain the hyperplane through point 2 and conclude that the solution must lie above this hyperplane also. The union of the doubly and triply cross-hatched areas now represents a measure of the subset of the feasible set which must contain the solution.

Again, we choose point 3 such that it lies above all previous hyperplanes, and we obtain the triply cross-hatched area as a measure of our progress. Note that each successive area is smaller than the preceding area, and only at the solution will this area be equal to zero. Note also

that as long as we choose each successive point such that it lies above all previously generated hyperplanes, we are assured that each successive area will be less than the previous area. Thus, this area which is a function of the generated points provides us with a good descent function. Minimization of this function over all points in the feasible alternative set is equivalent to solving the optimization problem of interest.

With this type of descent function there are many ways to choose the next iteration point, including randomly selecting a point which lies above the hyperplanes. However, the iteration with relaxation approach could be employed with the task of selecting a suitable relaxation coefficient governed by this alternate descent function.

The significance of this alternate descent function is that no further information is required from the decision-maker in order to search for a proper relaxation coefficient.



VIII. EXAMPLE OF ITERATION
 WITH RELAXATION
 (INITIAL RELAX. COEFF. = 0.5)

Example of Iteration with Relaxation

In order to illustrate how the iteration method works, the result of an example simulated on the computer is shown in this next figure. The example is a three-dimensional example so that the hyperplanes are now two-dimensional planes. The representation of these planes and the indifference surfaces and the complete alternative set on a two-dimensional figure becomes infeasible. Thus, the figure simply shows a representation of the feasible alternative set, and the iteration points in the $(x_3 = 0)$ plane. For this example the feasible alternative set corresponds to points which satisfy

$$\frac{x_1^2}{9} + \frac{x_2^2}{4} + x_3^2 = 100 \quad ,$$

when $x_3 = 0$ these points fall on the segment of an ellipse shown as the solid curve in the figure. For values of $x_3 > 0$, the x_1 and x_2 components will lie within the convex region bounded by the solid curve, and the two axes. The preference function was assumed to be

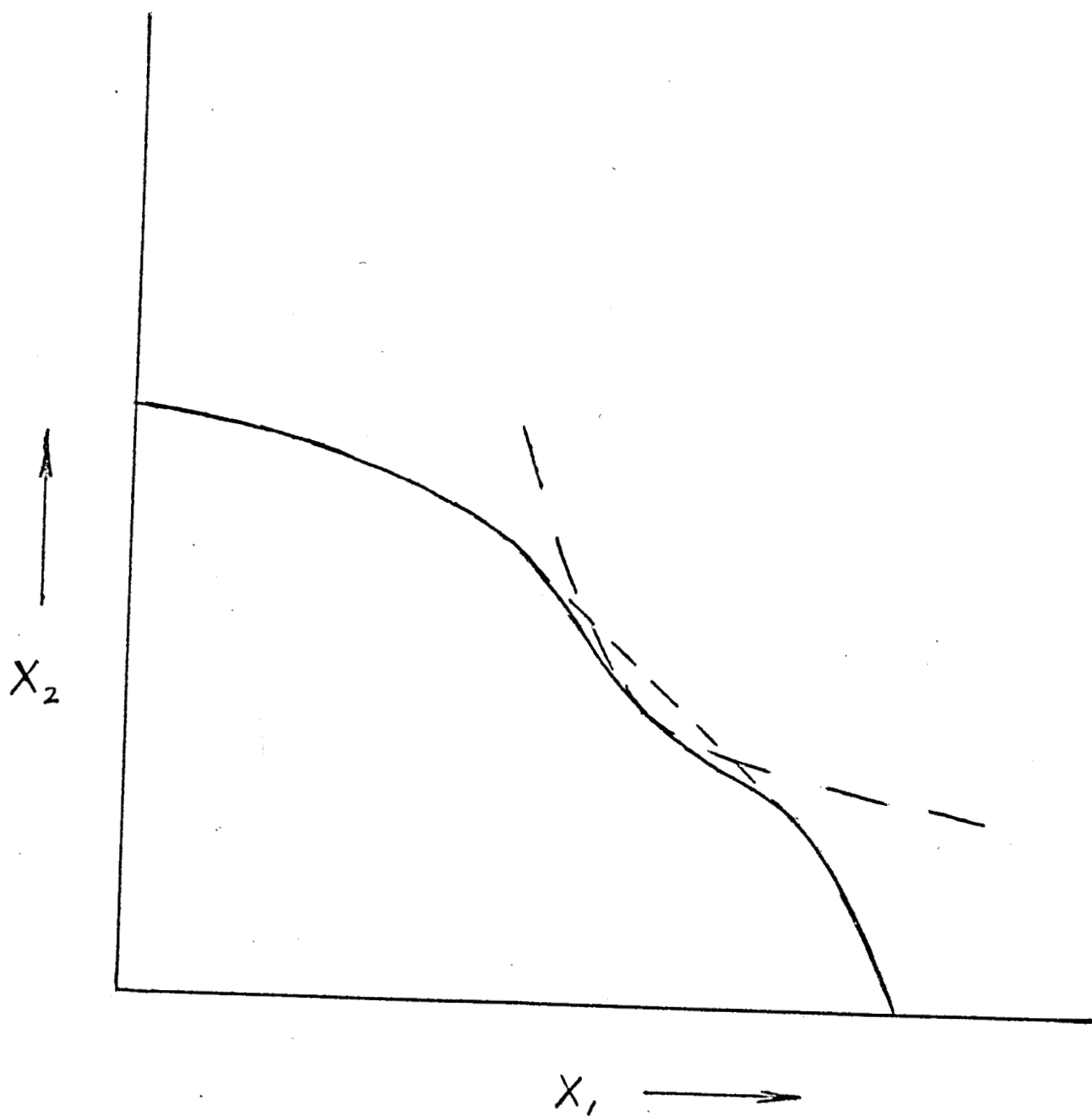
$$f(\tilde{x}) = x_1 \cdot x_2 \cdot x_3 \quad .$$

The x_1 and x_2 components of the solution are represented by the circled point in the figure. Iteration was arbitrarily begun at point 1 where $x_1 = x_2 = 9.5$, and x_3 satisfies the feasible set equation. It was also assumed that the resolution capability of the decision-maker in specifying his trade-off ratios was 0.001. Thus, the iteration was terminated when the trade-off ratio was within 0.001 of the feasible set tangent slope.

From the discussion thus far of iteration with relaxation, it was implied that at each iteration an initial relaxation coefficient of 1.0 be used. However, this is an area where the decision analyst can use his prior information to select some initial relaxation coefficient between 0.0 and 1.0. If it seems that the decision-maker's preference function is very linear, an initial relaxation coefficient at or near 1.0 would be appropriate. On the other hand, a high degree of nonlinearity would indicate an initial relaxation coefficient closer to 0.0. A neutral initial relaxation coefficient might be 0.5. This value was employed in the example.

The result of the simulated computer run was that five iterations were required to reach the solution. The difference between the fourth and fifth iteration and the solution was too small to show up in scale of the figure. Actually, the resolution limit of 0.001 is too low to be realistic, and a higher resolution figure might have been employed to stop at iteration 3, which is probably close enough to the solution.

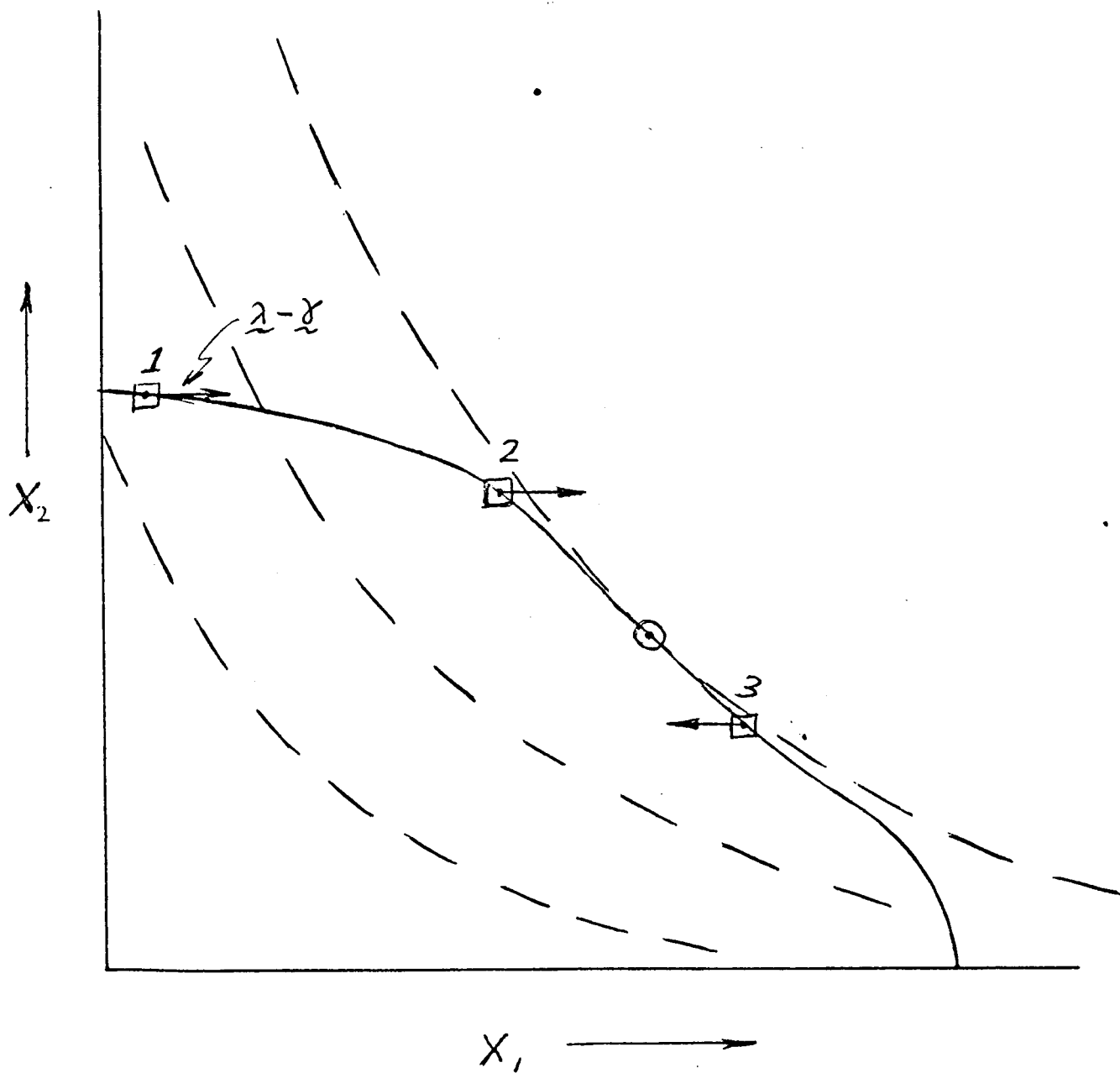
It is interesting to note that this example would have oscillated if relaxation were not employed. If an initial relaxation coefficient of 1.0 were used, about 11 iterations would be required. It is also interesting to note that, for this example, the relaxation coefficient never had to be reduced below 0.5 to obtain a decrease in the descent function when either the negative preference function or the hyperplane generated area function was employed as the descent function.



IX. GAPS

Gaps

Another special problem that might be encountered is the problem of gaps. A gap occurs when the feasible set is no longer convex as shown in this figure. The region of the gap is the region where the boundary of the feasible alternative set does not correspond to the convex hull of the set. The convex hull in the gap is shown by the dashed straight line in the figure. The dashed curved line represents the indifference curve defining the solution point. Note that for the two-dimensional example shown the solution lies in the gap. The difficulty is that the solution to any of the sub-optimization problem will not fall in the gap even if the starting point is selected in the gap. Thus, the iteration method will not work in this case. Note, however, that if a gap existed but the solution did not lie in the gap, the iteration method would work.



X. COORDINATE DESCENT
METHOD

Coordinate Descent Method

One method which would be capable of obtaining a solution in a gap is called the coordinate descent method. The method is a coordinate descent method in the context of problems with greater than two dimensions. For an n -dimensional problem, the idea is to choose a starting point, and keeping all but two coordinates fixed, search one of these coordinates for a more preferred point, and determine the final coordinate from the feasible set constraint. A new coordinate direction is then selected and the search is continued along the direction of the new coordinate. The two-dimensional problem represented in the next figure illustrates a typical coordinate search. In this case the search direction corresponds to the x_1 coordinate direction, and x_2 is uniquely determined for each value of x_1 by the feasible set constraint.

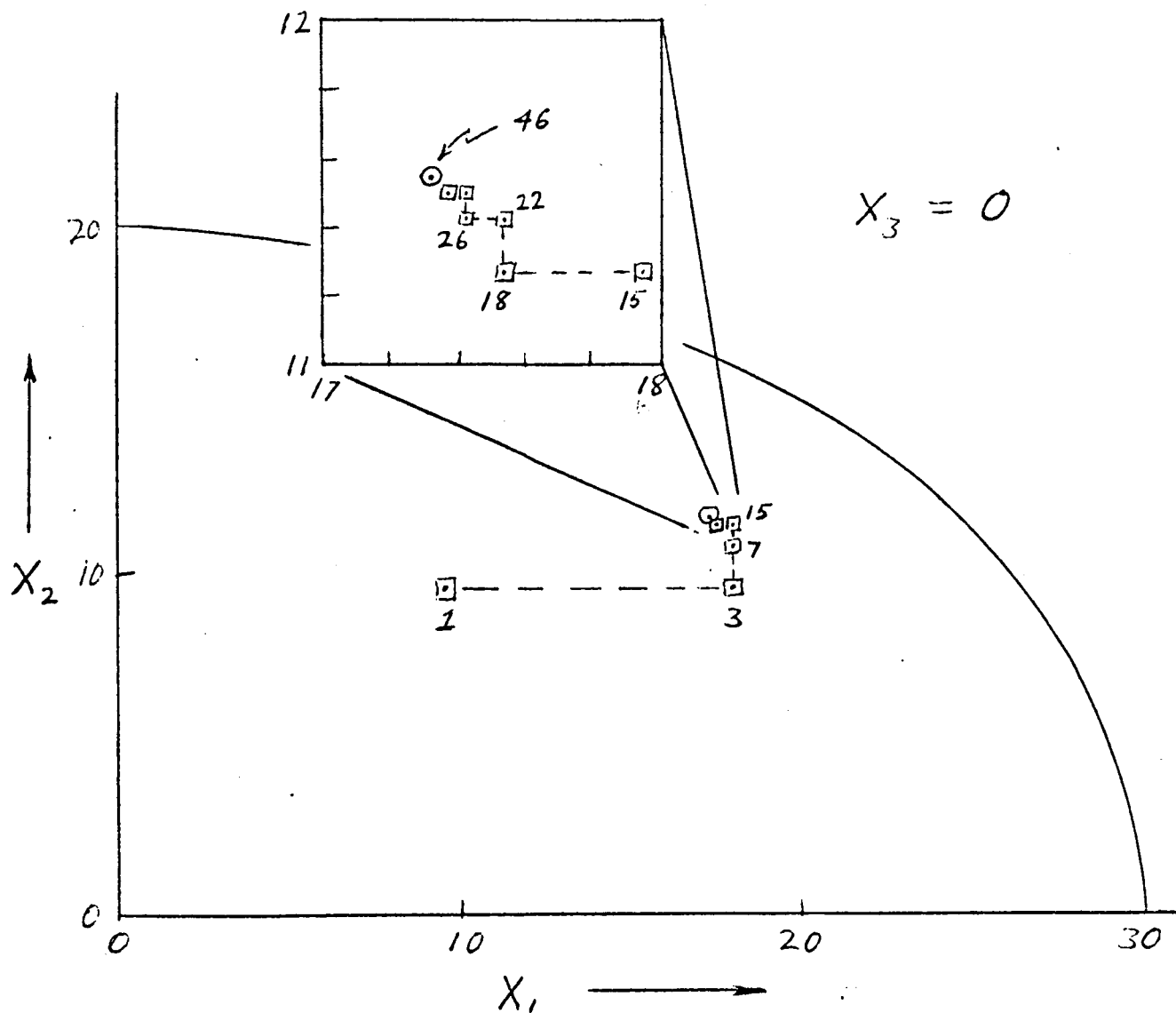
One feature of the coordinate descent method is that it can be implemented with much less information from the decision-maker per iteration. However, the cost of this advantage is an increase in the number of iterations required.

The coordinate descent iterations proceed as follows. At the starting point (point 1 in the figure) the decision analyst determines the slope of the feasible alternative set and then asks the decision-maker whether his trade-off ratio at that point is greater than or less than this slope. If it is neither, then we are at the solution (in higher dimensional problems we are not necessarily at the solution when this happens, but this event signals that a new coordinate direction should be tried). When the trade-off ratio is greater than the slope of the feasible set, we know that any preferred point must lie in the direction of the positive x_1 coordinate (as shown by the direction of the arrow at point 1). If the trade-off ratio were less, a search in the opposite direction would be indicated. Any technique might be employed in selecting the value of the x_1 coordinate for point 2 including an arbitrary selection. In any case, the procedure is repeated at point 2 and the result is that points preferred to point 2 must lie to the right, while all points to the left are less preferred.

Thus, in this case point 2 must be preferred to point 1.

In a higher dimensional problem we have a choice at point 2 either to find a more preferred point along the same coordinate, or to change the coordinate direction of the search. However, in the two-dimensional problem shown we continue by selecting point 3 somewhere to the right of point 2. At point 3 we find that we must reverse the direction of the search so that point 4 must lie between points 2 and 3. Eventually we must stop near the solution.

Note that if the order of selection of points 2 and 3 were reversed and if we had a higher dimensional problem, we could not have changed the direction of search after selecting the second point, since we would not know whether it was preferred to the first point or not. After the third point (this now corresponds to point 2 in the figure) we can change direction. Thus, the minimum condition to allow a change in search coordinate direction is that the direction of search along the old coordinate at the time we wish to change coordinates must be the same as the direction of the search (i.e., the positive or negative direction) from the starting point. This insures an increase in preference when we change coordinates.



XI. EXAMPLE OF COORDINATE
DESCENT METHOD

Example of Coordinate Descent Method

The same three-dimensional problem solved by computer simulation using the iteration method with relaxation was solved using the coordinate descent method. The result is shown in this next figure. The two possible coordinate directions correspond to the x_1 and x_2 coordinate directions. The solution point is circled, and only those iteration points corresponding to changes in coordinate directions are shown and numbered. Point 7 was a special case where an attempt was made to change coordinate directions but it was discovered that only a continuation in the same direction could result in improvement.

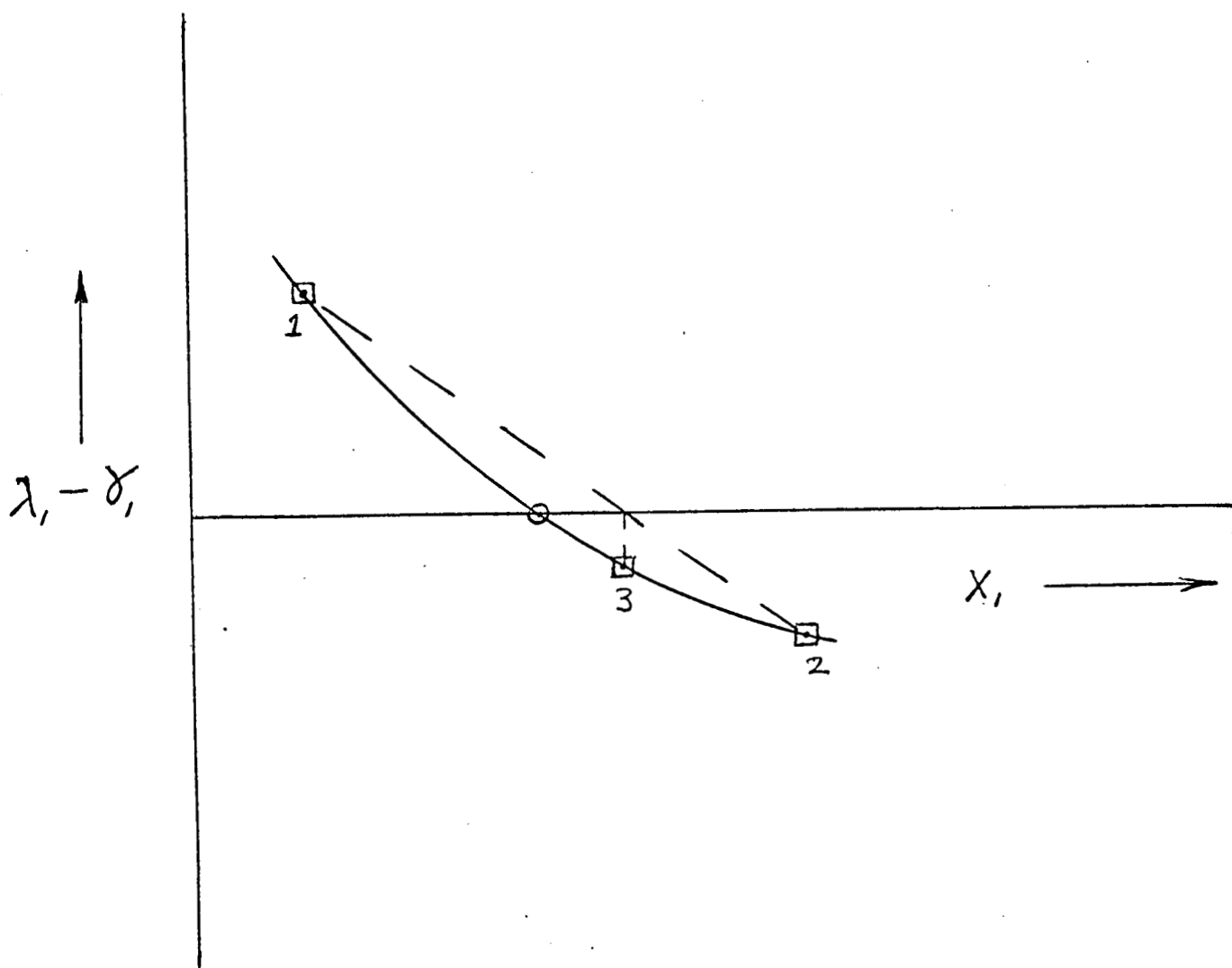
Under the same stopping rule as previously discussed, 46 iterations were required to reach the solution. Again, note that a more realistic stopping rule would probably stop us at point 15, which may be close enough to the true solution. In terms of the number of trade-off questions required to be asked of the decision-maker for each of the two methods discussed thus far, it should be noted that one iteration of the iteration method with relaxation requires two trade-off questions (i.e., between x_3 and x_2 , and between x_3 and x_1) whereas one iteration of the coordinate descent method requires only one trade-off question. Thus, in comparing the two methods in terms of number of trade-off questions, we should multiply the iteration with relaxation results by a factor of two.

We see that with the decision-maker's resolution threshold of 0.001, the coordinate descent method requires 4.6 times as many trade-off questions. On the other hand, if we had stopped sooner as discussed above, the coordinate descent method would require only 2.5 times as many trade-off questions. Thus, the coordinate descent method appears to have a slower rate of convergence as the solution is approached.

It should be especially noted that although the coordinate descent method requires more trade-off questions, the questions should require less effort by the decision-maker to answer. Thus, in terms of the decision-maker's cost in time and effort, it is not clear which method is better. One approach might be to combine the two methods by switching at some point like point 7 from the coordinate descent method to the

iteration method with relaxation. This last comment only applies if the solution does not lie in a gap.

Finally, the typical staircase pattern that develops after a number of iterations when using the coordinate descent method suggests that a search direction aligned somewhere between the two coordinates might be selected at some point to provide a more efficient search.



XII. SECANT METHOD

Secant Method

Another method that can be applied to the solution of the multi-attribute problem is the secant method. The secant method is capable of finding the solution even when the solution lies in a gap. The secant method is motivated by considering the difference between the trade-off ratio and the feasible alternative set as a function of the outcome variable x_1 . The remaining outcome variable x_2 is determined by the feasible set constraint. In the figure, the trade-off ratio is designated λ_1 and the feasible set slope is designated γ_1 . Thus, the function we are interested in is $\lambda_1 - \gamma_1$. The curved solid line in the figure represents this function. The solution we are looking for is the point where $\lambda_1 = \gamma_1$, or $\lambda_1 - \gamma_1 = 0$. This point is circled in the figure.

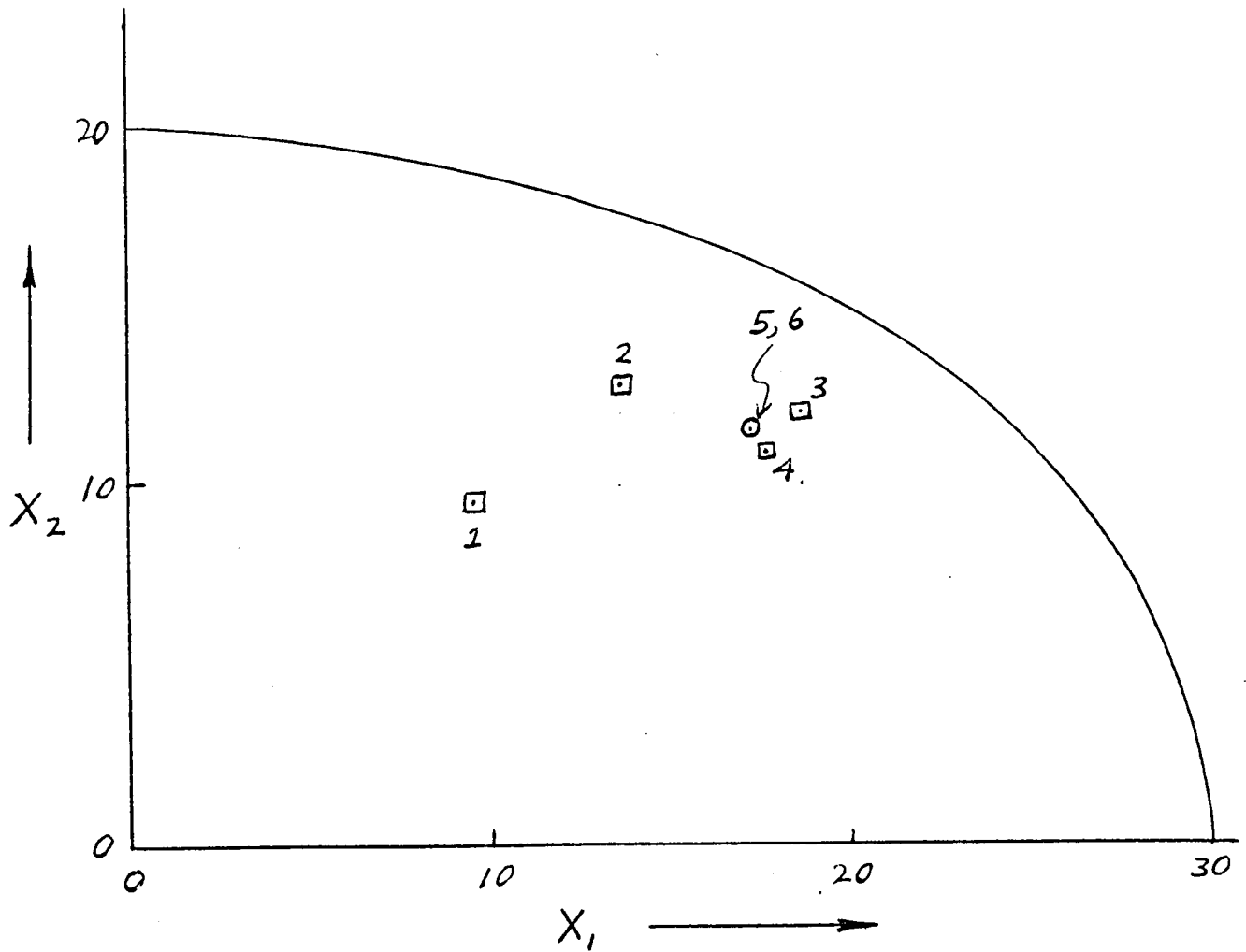
The secant method requires two points to initiate it, such as points 1 and 2. The values of x_1 at these two points can be selected by any other technique. The values of $\lambda_1 - \gamma_1$ at these points are obtained as before by asking the decision-maker his trade-off ratios at these points. Given points 1 and 2 we temporarily assume that the nonlinear curve is approximated by the linear curve (i.e., the dashed line). This approximation indicates the solution might be point 3. Since the value of $\lambda_1 - \gamma_1$ at point 3 is not zero, we continue our search by assuming a linear approximation of the function using points 2 and 3. As we approach the solution, the linear approximation becomes better and we should converge to the solution.

Note that at each stage we have two points which form a basis for the linear approximation. As we obtain a new point, we introduce it into the basis, and are required to drop one point from the basis. Here we have a choice. In the example we could have dropped either point 1 or point 2. We are motivated to drop point 1 by the fact that $\lambda_1 - \gamma_1$ at point 1 is larger than $\lambda_1 - \gamma_1$ at point 2. If this had been reversed we might choose to drop point 2 and form our linear approximation using points 1 and 3. Another approach would be to drop the oldest point (i.e., point 1). There is no general statement that can be made at this point about which is the better strategy since this

would be strongly influenced by the particular nonlinear function with which we were working. However, in some cases, as in this example, the two strategies are equivalent.

In any case, in order to insure convergence we require a descent function as before. One possible descent function is the magnitude of $\lambda_1 - \gamma_1$. If this descent function is chosen and if each successive iterate is chosen to decrease this descent function, the two strategies for updating the basis will be equivalent. However, it is necessary to modify the method slightly to insure being able to obtain a new point with an improvement. We do this by selecting as our next iterate some point which is a linear combination of the point determined by the linear approximation and the last point such that the descent function is decreased (i.e., in the example, the next iterate after point 2 might be some linear combination of points 2 and 3). Again, this is analogous to the search required for the relaxation coefficient, and it suffers from the same disadvantages. As before, we can employ the hyperplane generated area function as an alternate descent function and avoid most of these difficulties.

The secant method has been discussed in a two-dimensional problem context but can be readily extended to n-dimensional problems. In such cases the optimization problem can be reduced to solving (n-1) nonlinear simultaneous equations.



XIII. EXAMPLE OF SECANT
METHOD

Example of Secant Method

The same three-dimensional example solved previously has been solved by the secant method and the results are indicated in this next figure. Note that the secant method requires six iterations to reach the solution. This is one more iteration than the basic iteration method with relaxation. However, as stated before, the secant method is applicable even in cases where the solution lies in a gap. Again, we can note that a more realistic stopping rule might stop at point 4 which may be close enough to the solution.

XIV. SUMMARY OF RESULTS

(AVERAGE NUMBER OF ITERATIONS
REQUIRED OVER 5 RUNS)

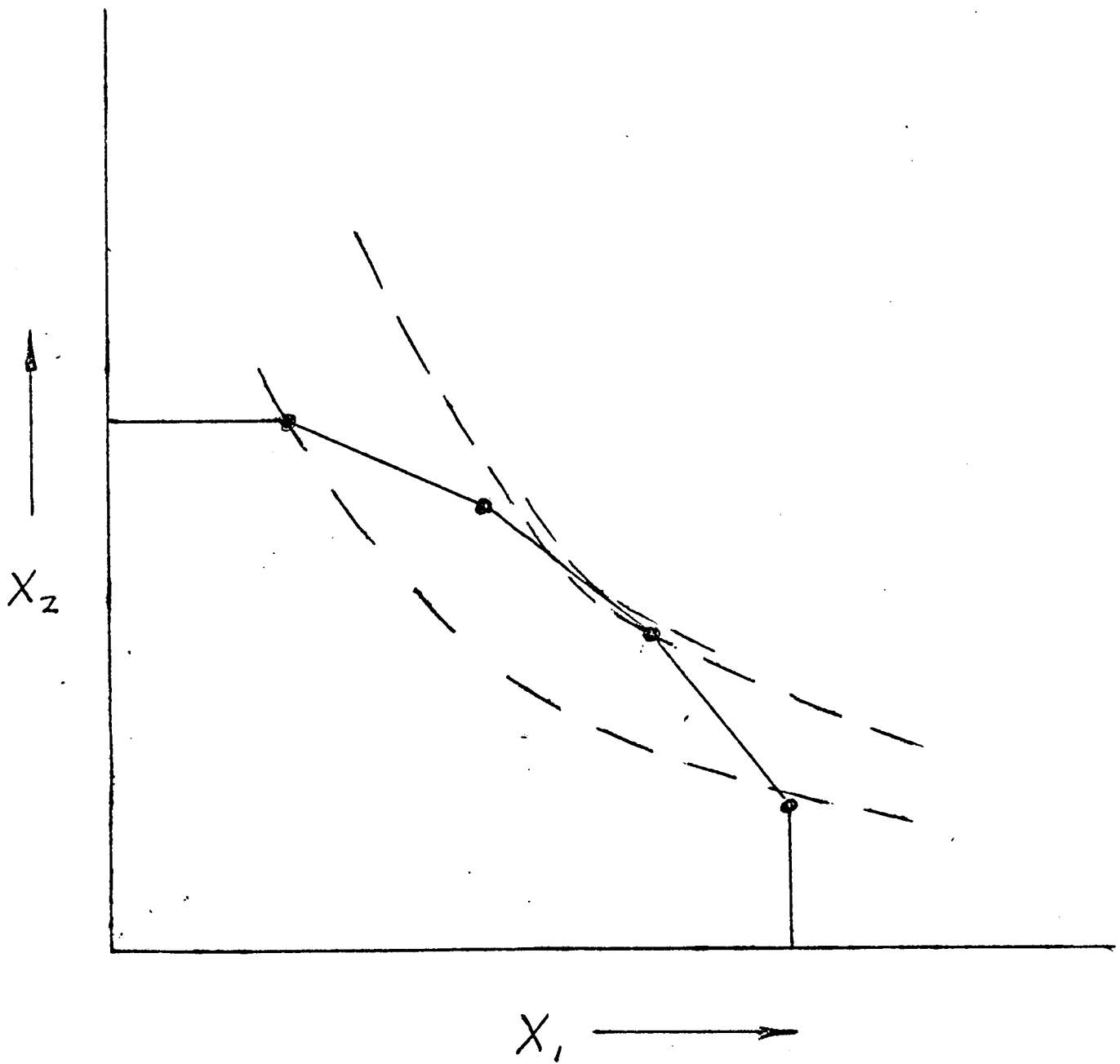
CASE	TRADE-OFF	COORDINATE DESCENT	ITERATION WITH RELAXATION	SECANT
	RESOLUTION			
1	0.001	67	5	6.6
2	0.001	90+	5.2	7.8
1	0.01	40	4.6	6
2	0.01	58	4.6	6.8

Summary of Results

A second example problem was constructed and simulated on the computer which differed from the first example only in the preference function. The new preference function was similar in form to the old but differed in the values of certain parameters. The first example will be called case 1 and the second, case 2. Each case was solved by each method five times using a different starting point each time. The average number of iterations required for each of two alternate thresholds was computed and the results are tabulated in this next figure.

The upper two rows of the table show the results when the resolution threshold is 0.001 and the lower two rows when it is increased to 0.01. Note that the secant method requires about two more iterations than the iteration method with relaxation. To compare these results to the coordinate descent method we should multiply them by a factor of two. The coordinate descent method requires from 40 to 100 iterations depending on the threshold and the case. However, notice that the lower the resolution threshold, the more rapidly the required number of iterations increases as compared to the other two methods. Due to the unique advantage of the coordinate descent method concerning the low amount of information required per iteration, these results suggest a combined approach using the coordinate descent method to get to the neighborhood of the solution, and switching to the iteration method with relaxation or the secant method to converge to the solution.

Further research work will be concerned with how the decision analyst can use decision analysis techniques to decide at each cycle of the iteration whether to stop or continue, and if to continue, which method to employ for the next iteration.

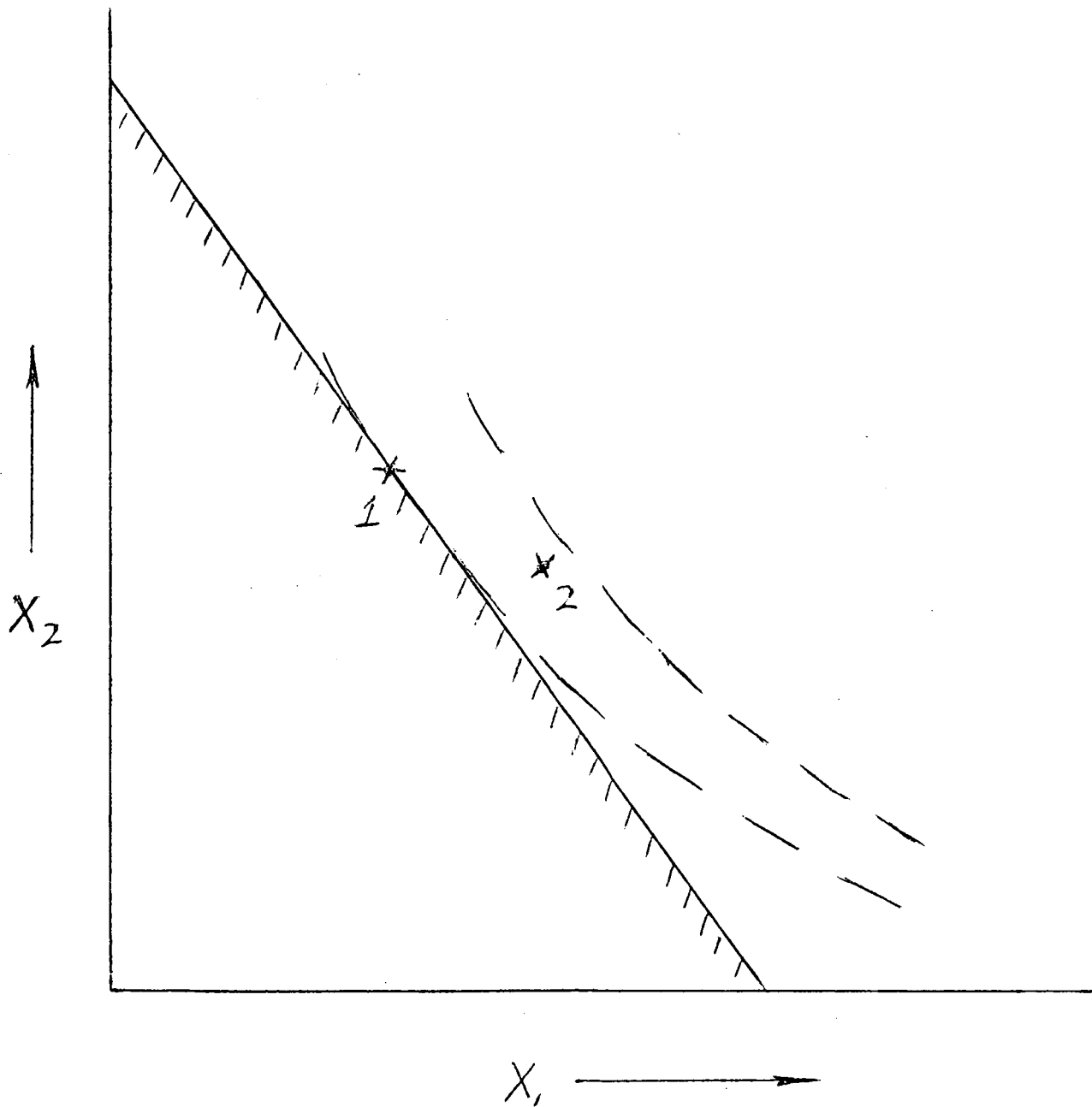


XV. DISCRETE ALTERNATIVES

Discrete Alternatives

In general, the decision problem may not be formulated with a continuum of alternatives and outcomes as discussed up to this point. The feasible set may simply consist of several discrete outcomes (corresponding to discrete alternatives) as indicated by the black dots. We can handle this problem as a continuous problem by joining the points by straight lines to form the convex hull of the feasible set. In such a case, we may get one of two types of results. First, we may converge to one of the discrete points in which case we will have the true solution. (This case is not shown in the figure.) A more likely case is shown in the figure where the solution to the convex hull problem lies on the line between two of the discrete points, and our methods will converge to this point, which does not belong to the discrete feasible set. However, we will have reduced the subset of the feasible set containing the solution to two discrete points. (For the n -dimensional problem this subset will contain at most n points.)

At this point we cannot find which point of the subset is the solution. There are several approaches which can be pursued to make the final selection. First, the member of the subset lying closest to the solution to the convex hull problem can be arbitrarily taken as the best approximation to the solution. A second approach is to ask the decision-maker to make a final selection based on his perceived preferences. A third approach is to attempt to construct a new discrete alternative point in the neighborhood of the convex hull solution and determine by a repeat of one of the methods whether it is a solution or whether a new convex hull solution is obtained which does not belong to some subset of discrete points. This process can be repeated as often as is economical in order to get closer to the solution.



XVI. DISCRETE ALTERNATIVE
ITERATION

Discrete Alternative Iteration

The previous discussion suggests an approach that might be used when the construction of alternatives is itself a costly process which we would like to minimize. In such a case, we might want to follow the approach illustrated in this next figure. This approach involves embedding not only the preference assessment task, but also the alternative construction task in the optimization process.

Referring to the figure, consider constructing a single alternative represented by point 1 in the outcome space. At point 1 we can then ask the decision-maker what his trade-off ratio is. This information is represented by the solid line through point 1. As discussed before, this line is a hyperplane such that all points in the space to the lower left of it are less preferred than point 1. If there is some point preferred to point 1 it must lie to the upper right of this hyperplane. This suggests that a new alternative should be sought which lies to the upper right. If the analyst can convince himself that no new alternative point can be constructed which lies in this region, he then knows that point 1 is the optimum. However, if an alternative can be constructed such that the corresponding outcome is a point such as point 2, the process can be repeated until a solution is obtained or the region where a solution must lie has been sufficiently reduced.

This final idea will be further developed in the next phase of my research.

SOME COMMENTS ON THE FIRST PASS OF THE LIMITED
MANNED SPACE FLIGHT PILOT STUDY

By

RICHARD D. SMALLWOOD

This memorandum is a commentary on the general approach by the Mission Analysis Division pilot study of a limited manned space flight program. Particular emphasis is given to the results of the first pass of that pilot study as documented in the memorandum of March 20, 1970. This memorandum presents three recommendations for the next phase of this pilot study:

- a. The evaluation of outcomes should maintain a careful distinction between outcome variables and value variables.
- b. The value structure for the various environmental elements should be carried out as a separate activity from the description of outcomes. These two results should then be combined to evaluate the relative worth of the outcomes.
- c. The analysis should be expanded somewhat to include some of the major uncertainties associated with each alternative. In particular, some consideration should be given to structuring more than just the nominal outcome for each of the program alternatives.

The following is a brief outline of one scheme for accomplishing the evaluation phase of the pilot study. The scheme is similar in philosophy to the one outline by Roger Arno in his memorandum, although it is somewhat simpler in detail.

The main objective of the evaluation phase is to assign some measure of relative worth to the outcomes associated with each alternative program. These worth measures can be qualitative statements, ordinal preferences, formal quantitative measures, or any combination of these. Figure 1 illustrates the general structure that is used to carry out this evaluation process. On the left-hand side are the

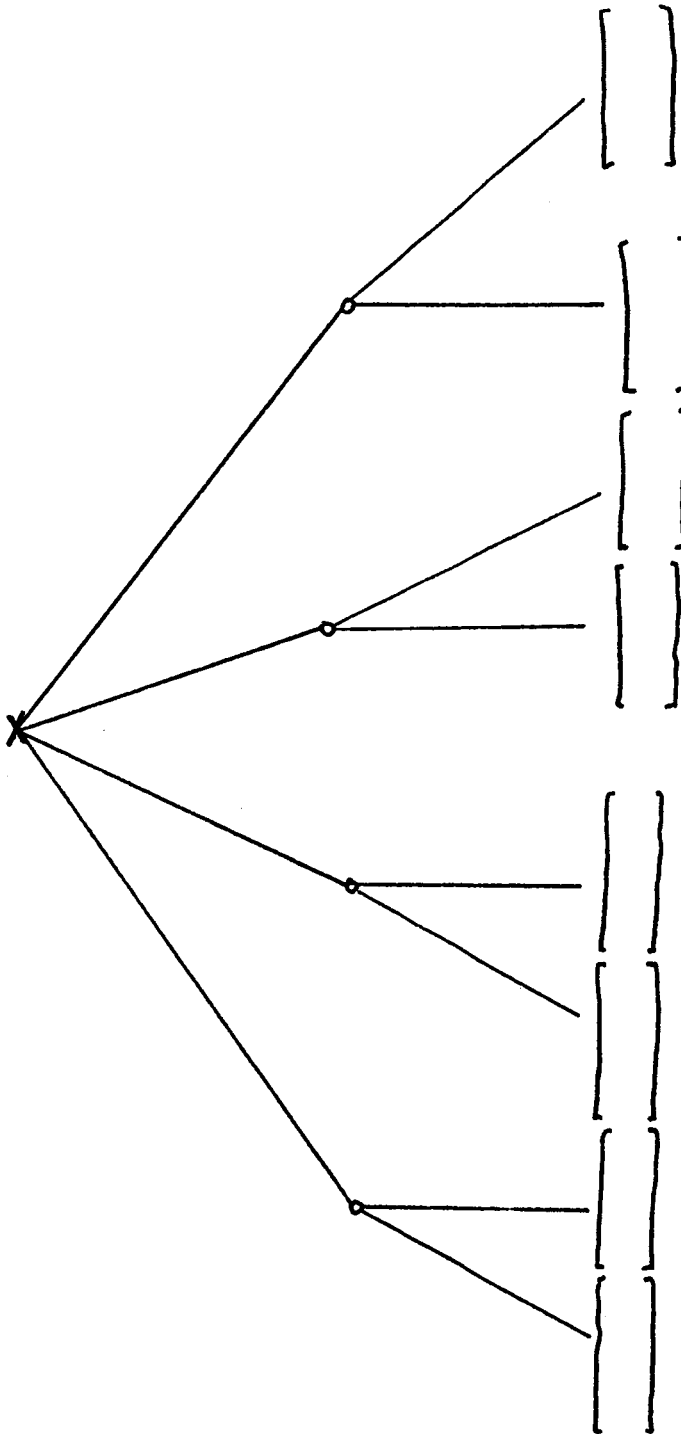


Figure 1 Program decision tree

four alternative programs; the problem is to decide which of these programs should be conducted by the agency. The second series of branches on this decision tree represent the outcomes associated with each program. Thus, in the first pass only a single outcome for each of the programs was included, although later passes may consider a more detailed structuring of the outcomes. In any case, these outcomes generally represent a rather complex series of events that have occurred as a result of the decision to engage in that program. The far right-hand brackets in Figure 1 represent the set of numbers necessary to specify each outcome. The dimensions for this set of numbers will be called outcome variables in this memorandum. The intent here is to specify each outcome in very specific technological terms. Thus, from Table 1 of the March 20 memo, the following outcome variables could be used: number of foreign nations cooperating in the program, launch date for the space station, maximum gap in manned space flight missions, average number of manned space flight launches per year, number of scientists in orbit, specific quantities of scientific data gathered, services rendered to various segments of our society, competitive victories over the U.S.S.R., and new technological capabilities for future exploitation. Thus, we can imagine constructing for each outcome under consideration a set of numbers or statements that essentially defines the outcome.

The next problem is to decide what segments of NASA's environment should be considered in evaluating each of these outcomes. The left-hand column in Table 1 from the March 20 memo is a good list of these elements. In attempting to describe the response of each of these environmental elements to the outcomes of the four programs, the first step is to encode the response of each environmental element to each of the outcome variables. Thus, we can imagine a matrix as shown in Figure 2 in which each cell represents the response of that environmental element to the outcome variable. The technique used to record the entry in each cell of the matrix

is a decision that must be made by the analysis staff. A technique consistent with the March 20 memo would be a series of pluses and minuses with a plus denoting a favorable response: a minus, a negative response; a zero, indifference; and a blank, ignorance. The important point is that the entries in this matrix should be constructed more or less independently of the alternative programs and outcomes under consideration.

Once this has been accomplished we can imagine using this matrix to compare the outcome variables for each outcome and thus arrive at a composite evaluation of each outcome for each environmental element. This process is illustrated in Figure 3. The advantages of a scheme such as this are that the value structuring of each environmental element is carried out separately from the evaluation of the outcomes. Maintaining this distinction should clarify some of the steps in the evaluation process.

In some cases it may be necessary to expand the list of environmental elements in Figure 2 to account for several aspects of a single element. For example, it may be desirable to decompose the European element into two components, its attitude toward U.S. prestige and its attitude towards cooperation with the United States.

By maintaining this distinction between environmental elements and outcome variables and by separating the evaluation of environmental preferences from evaluation of outcomes, some of the evaluations in Table 1 of the March 20 memo can be brought into sharper focus. For example, at the present time it would seem more reasonable to eliminate the entry "enhances national prestige" as an outcome variable and incorporate it instead as one of the national goals. In addition, it should now be possible to decide by looking at the completed matrices in Figures 2 and 3 to decide whether or not a plus or minus in Table 1 represents a failure on the part of the outcome to supply that outcome variable or a negative view on the part of

OUTCOME VARIABLES

ENVIRONMENTAL ELEMENTS

	No. of foreign nations cooperating	Launch date for space station	Max. gap in manned space flight missions	...	Competitive victory over USSR	New tech. capabilities for future
USSR						
Europe						
...						
OMSF						
NASA admin.						
...						
...						
National ideals						

Figure 2 Value structure matrix

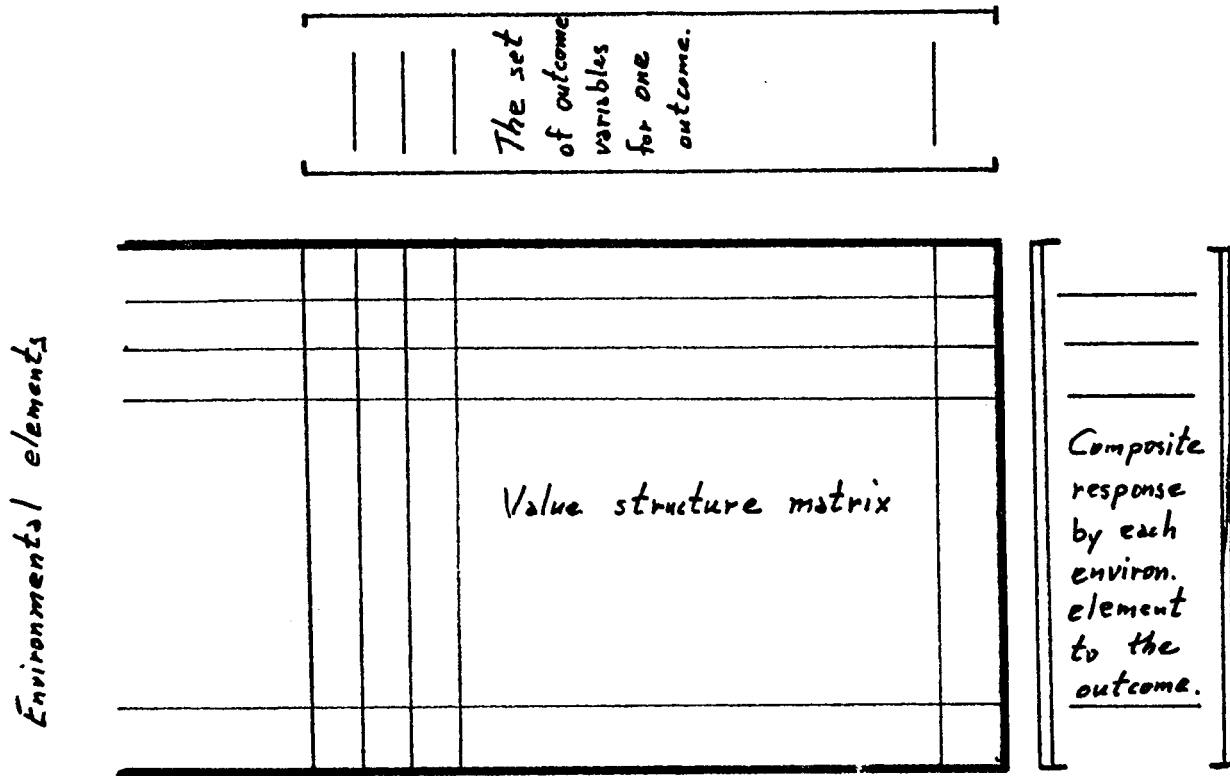


Figure 3 The evaluation process.

the environment toward that particular outcome variable.

The third recommendation for the second pass in the pilot study suggests that more than one outcome for each alternative be considered to account for uncertainty in the outcome of a program. The reasons for this recommendation are:

- a. Uncertainty in the outcomes of a program are an important reality that should be considered in any planning process.
- b. Some attempt should be made to introduce analysis of uncertainty in the long-range planning process within NASA.
- c. It will be instructive on the part of our MAD group to gain experience in this part of the planning process.

The consideration of uncertainty is of course not an alien topic to NASA -- at least at the operational level -- as witness the various contingency plans and redundancy systems built into NASA's hardware. It is strange then that an agency with such an awareness of the vagaries of fortune should carry out its planning on a strictly nominal basis. This is not a recommendation to implement an exhaustive ultra sophisticated treatment of uncertainty for the pilot study. Rather it is a recommendation to include a few serious outcomes for each program that are not necessarily the ideal or nominal ones.

Development of the Mathematical Structure of the Multi-Level,
Multi-Attributed Problem

At this point of my research into the methodology for determining the worth of multi-attributed space program alternatives with respect to groups having a high degree of common values and concerns, it would be useful to consider the general mathematical framework of the problem so that various approaches to the problem can be outlined and their inter-relationship made visible. In particular, this will also allow exposition of where the multi-level attribute approach which I have been advocating fits into the overall picture.

The mathematical development starts from the general form of the differential equation describing a preference function of an individual or group, and develops the implications of various assumptions that can be made concerning this differential form.

Let us start by assuming that any given alternative can be encoded into a vector of outcome variables which are continuously quantifiable over some interval. Designate this outcome vector as

$$\underline{x} = (x_1, x_2, \dots, x_n) \quad (1)$$

Also assume that there exists a continuous, differentiable worth function which is a real valued function of the vector \underline{x} , and which encodes an individual's or group's preferences for a given outcome. We designate this worth function as

$$w = w(x_1, \dots, x_n) = w(\underline{x}) \quad (2)$$

The differential form of Eq. (2) can be expressed as

$$dw = \sum_i \frac{\partial w}{\partial x_i} dx_i \quad (3)$$

In general, the partial derivatives in Eq. (3) are also functions of the vector \underline{x} . By defining

$$\frac{\partial w}{\partial x_i} \equiv \lambda_i(\underline{x}) , \quad (4)$$

we can rewrite Eq. (3) as

$$dw = \sum_i \lambda_i(\underline{x}) dx_i \quad (5)$$

Since Eq. (5) is an exact differential we can obtain w by the following line integral form.

$$w = \sum_i \left[\int_{c_i}^{x_i} \lambda_i(\underline{s}) \left| \begin{array}{ll} s_j = c_j & \text{for } j > i \\ s_j = x_j & \text{for } j < i \end{array} \right. ds_i \right] \quad (6)$$

where the c_j 's are arbitrary constants, and the s_j 's are dummy variables of integration.

Without any further knowledge of the functional form of $\lambda_i(\underline{x})$, we do not find Eq. (6) particularly useful. We thus seek to find reasonable assumptions that can be made concerning $\lambda_i(\underline{x})$ and derive the implied functional form of w and means to test the reasonableness of the underlying assumptions.

The first assumption we can make is that

$$\lambda_i(\underline{x}) = a_i , \quad \text{a constant} \quad (7)$$

This is a very restrictive assumption which states that the worth function is linear in each of the outcome variables and that the incremental change of the worth function due to a change in one outcome variable is independent of the value of all other outcome variables. Equation (7) implies that

$$w = \sum_i a_i x_i, \quad (8)$$

where the constant of integration has been assumed to be zero.* Any problem for which this assumption is valid implies that n weighting coefficients with respect to the n outcome variables need be evaluated to obtain w . Alternatively, we can write Eq. (8) as

$$w = a_n x_n + \sum_{i=1}^{n-1} b_i x_i \quad (9)$$

In this form we need only evaluate the $n-1$ weighting coefficients b_i , since we are only interested in relative differences in worth between alternatives and may thus assume $a_n = 1$.

Although we cannot expect this assumption to be valid over all the outcome variables specified for a given decision problem, we may find that it holds for some subset of the outcome variables. When such is the case, the problem can be decomposed and reduced in complexity in the following manner.

Let the index set be I , and consider a partition of this index set into J and K , where $J \cup K = I$ and $J \cap K = \emptyset$, the null set. Assume that

$$\lambda_i(\underline{x}) = a_i \quad \text{for } i \in J \quad (10)$$

and

$$\lambda_i(\underline{x}) = \lambda_i(\underline{x}') \quad \text{for } i \in K \quad (11)$$

* Since we are only interested in the difference in worth between alternatives, we need not concern ourselves with this constant.

where

$$\underline{x}' = (x_{i_1}, x_{i_2}, \dots, x_{i_m})$$

and

$$i_k \in K, \text{ and } m \leq n.$$

Then we can write

$$w = \sum_{i \in J} a_i x_i + \sum_{i \in K} \left[\int_{c_i}^{x_i} \lambda_i(\underline{s}') \left| \begin{array}{ll} s_j = c_j & \text{for } j > i \\ s_j = x_j & \text{for } j < i \end{array} \right. ds_i \right] \quad (12)$$

Thus, the problem decomposes into deriving $n-m$ weighting coefficients, and reducing that portion of the problem for which more sophisticated techniques are required to an m -dimensional problem.

Another assumption one might make is that

$$\lambda_i(\underline{x}) = a_i \gamma_i(x_i) \quad (13)$$

We can then write

$$w = \sum_i a_i \int_{c_i}^{x_i} \gamma_i(x_i) dx_i \quad (14)$$

Let

$$\int_{c_i}^{x_i} \gamma_i(x_i) dx_i = u_i(x_i) \quad (15)$$

Then

$$w = \sum_i a_i u_i(x_i) \quad (16)$$

Equation (16) indicates that the worth function has an additive representation, and has been decomposed into the problem of deriving n real valued functions of real variables and n weighting coefficients. Actually, as before, only $n-1$ weighting coefficients are required. Presumably, the derivation of the n functions is feasible, and several methods employing worth scoring or choice of lotteries have been proposed for this problem (see Miller [1] and Raiffa [2]).

The necessary assumption implies an independence of the effect of an outcome variable, x_i , on the worth function, w , from the effect of another outcome variable, x_j , for $i \neq j$.

A finer structure useful for evaluating the weighting coefficients can be incorporated into Eq. (16) in the following manner. Partition the index set I into some number of partitions. Let us assume we partition into two sets J and K . The rationale for the partition will be subsequently discussed. However, regardless of the meaning of the partition, we can then write

$$w = a_{i_j} \sum_{i \in J} \frac{a_i}{a_{i_j}} u_i(x_i) + a_{i_k} \sum_{i \in K} \frac{a_i}{a_{i_k}} u_i(x_i) \quad (17)$$

The form of Eq. (17) indicates that the derivation of the weighting coefficients can be decomposed into two (or more) levels. The rationale for the partition is that groups of outcome variables may support some attribute of the alternatives which is defined at a higher level. Thus, at each level in such a hierarchical structure, one can think about trade-offs between attributes. Note that at each level the independent assumption between attributes must hold if the final outcome variables are to be independent.

Again, as before, it will not be realistic in a complex problem that the above assumption is valid for all outcome variables. In this case we might have a partition of outcome variables such that

$$\lambda_i(\underline{x}) = a_i \quad \text{for } i \in J \quad (18)$$

$$\lambda_i(\underline{x}) = a_i \gamma_i(x_i) \quad \text{for } i \in K \quad (19)$$

$$\lambda_i(\underline{x}) = \lambda_i(\underline{x}') \quad \text{for } i \in L \quad (20)$$

where \underline{x}' is defined as before by Eq. (11). Then we can write

$$w = \sum_{i \in J} a_i x_i + \sum_{i \in K} a_i u_i(x_i) + \sum_{i \in L} \left[\int_{c_i}^{x_i} \lambda_i(\underline{s}') \left| \begin{array}{l} s_i = c_i \text{ for } j > i \\ s_j = x_j \text{ for } j < i \end{array} \right. ds_i \right] \quad (21)$$

Equation (21) indicates a decomposition similar to that provided by Eq. (12).

In some cases a quasi-additive form may be assumed. The quasi-additive form for the two dimensional case (designating the components x and y) is given by

$$w(\underline{x}) = w_x(x) + w_y(y) + a w_x(x) w_y(y) \quad (22)$$

Here again, the task of assessing w has been decomposed into the tasks of assessing two functions on each of the real variables x and y . However, the weighting coefficient on the joint effects of w_x and w_y must now also be estimated.

The property that implies quasi-additivity is that x and y must be strongly conditionally utility independent (SCUI). The definition of SCUI is the following: x is SCUI of y if and only if the preferences between lotteries having consequences of the form (x, y^0) (where the second component y^0 is identical for all consequences) do not change if the common value y^0 is changed to y' (for any y^0 and y').

Thus, the quasi-additive form is useful since we may be able to determine if it is a good assumption by asking questions concerning preferences for lotteries. Raiffa [2] shows how w_x , w_y , and a can be determined.

One final form might be assumed which would be useful. This form is called the log additive form and is given by

$$\log [a + bw(\underline{x})] = \sum_i w_i(x_i) \quad (23)$$

for some constant a and b , $b > 0$. It is interesting to note that quasi-additivity implies log additivity (but not vice versa). Pollak [3] presents axioms which imply log additivity, and Keeney [4] presents suggestions for practical assessment. These topics will not be discussed at this time since my purpose is simply to summarize some of the useful forms for w that may be inferred.

When the simple additive forms are not appropriate to the particular problem (or to some reduced dimension of the problem), one approach is to go to iterative techniques such as proposed by Boyd [5]. Basically this approach is to take a linear form as an approximation to w in the neighborhood of one alternative outcome vector. One then determines whether the same alternative outcome is optimum with respect to the linear form.

If it is, the alternative will also be optimum with respect to the true form of w . The linear form approximation to w is then a good approximation with respect to the optimization problem at hand.

If the alternative outcome optimizing the linear form of w is not the same as the alternative outcome at which one obtained the linear form, a new linear form is obtained at the new alternative outcome point. This process is repeated until convergence is achieved. Convergence is not necessarily achieved in all cases, and Boyd outlines the conditions (which are generally not very restrictive with respect to most types of decision problems encountered) required for convergence.

Mathematically, if we designate the i^{th} approximation to w as \hat{w}^i and the alternative outcome at which it is assessed as \underline{x}^i , we can write

$$\hat{w}^i(\underline{x}^i) = \sum_j \left. \frac{\partial w}{\partial x_j} \right|_{\underline{x} = \underline{x}^i} x_j \quad (24)$$

or

$$\hat{w}^i(\underline{x}^i) = \left. \frac{\partial w}{\partial x_n} \right|_{\underline{x} = \underline{x}^i} \sum_j \left. \frac{\partial x_n}{\partial x_j} \right|_{\underline{x} = \underline{x}^i} x_j \quad (25)$$

Equation (25) shows that the only assessments to be made are the $n-1$ trade-off ratios between the j^{th} and the n^{th} outcome elements required in order to remain indifferent to any initial perturbation of the outcome vector.

At this point let us return to our original assumption that w is a function of the outcome variable vector \underline{x} (Eq. (2)). It may turn out for a given problem that the level of definition of outcome variables is too far removed from the level of definition of the values of a group to

allow assessment of the various functions indicated in the preceding development with a reasonable amount of effort. In this case, it may be possible to define intermediate levels of attributes which are supported by the next lower level of attributes, and which support the next higher level of attributes. Within this structure it may be easier to evaluate the stagewise dependence between adjacent levels of attributes. The resulting information can then be combined to derive a worth function on the outcome variables.

Mathematically, the development of this approach goes in the following manner. Consider the case where only one intermediate level of attributes has been interjected. We now assume that the worth is a function of the higher level of attributes, and each of these attributes is a function of the outcome variables. Thus, we are assuming

$$\underline{y} = (y_1, \dots, y_m), \text{ the intermediate level attribute vector,} \quad (26)$$

$$w = w(\underline{y}) \quad (27)$$

and

$$y_i = y_i(\underline{x}) . \quad (28)$$

These relations also imply

$$w = w(\underline{x}) \quad (29)$$

We now consider the differential form of w ,

$$dw = \sum_i \frac{\partial w}{\partial y_i} dy_i , \quad (30)$$

and the differential form of y_i ,

$$dy_i = \sum_j \frac{\partial y_i}{\partial x_j} dx_j \quad (31)$$

Combining Eqs. (30) and (31) and re-ordering terms, we obtain

$$dw = \sum_j \left[\sum_i \frac{\partial w}{\partial y_i} \frac{\partial y_i}{\partial x_j} \right] dx_j \quad (32)$$

We recognize the sum in the brackets as simply the partial of w with respect to x_j as given by the chain rule of the calculus for composite functions.

The partial derivatives in Eq. (32) as before are generally functions of vectors. To make this explicit let

$$\frac{\partial w}{\partial y_i} = \alpha_i(\underline{y}) = \alpha_i(\underline{x}) \quad (33)$$

and

$$\frac{\partial y_i}{\partial x_j} = \beta_{ij}(\underline{x}) \quad (34)$$

Equation (32) can then be written as

$$dw = \sum_j \left[\sum_i \alpha_i(\underline{x}) \beta_{ij}(\underline{x}) \right] dx_j \quad (35)$$

Since Eq. (35) is an exact differential we obtain

$$w = \sum_j \left[\int_{c_k}^{x_k} \sum_i \alpha_i(\underline{s}) \beta_{ij}(\underline{s}) \left| \begin{array}{l} s_k = c_k \text{ for } k > j \\ s_k = x_k \text{ for } k < j \end{array} \right. ds_j \right] \quad (36)$$

Let us assume that at the higher intermediate level

$$\alpha_i(\underline{x}) = \alpha_i(\underline{y}) = a_i, \quad \text{a constant} \quad (37)$$

and

$$\beta_{ij}(\underline{x}) = b_{ij} \gamma_{ij}(x_j) \quad (38)$$

These relations together with Eq. (36) result in the following equation

$$\tilde{w} = \sum_j \sum_i a_{ij} b_{ij} \int_{c_j}^{x_j} \gamma_i(s_j) ds_j \quad (39)$$

Let

$$\int_{c_j}^{x_j} \gamma_{ij}(s_j) ds_j = v_{ij}(x_j) \quad (40)$$

Then

$$w = \sum_i a_i \left[\sum_j b_{ij} v_{ij}(x_j) \right] \quad (41)$$

Equation (41) indicates that the worth function assessment problem has been decomposed into the tasks of deriving $m \times n$ real valued function of a real variable $(v_{ij}(x_j))$, $m \times n$ weighting coefficients (b_{ij}) and m weighting coefficients (a_i) . In practice, there will be some outcome variables and intermediate level attributes which are not significantly related. Thus, we will actually need to derive somewhere between n and $m + n$ functions $(v_{ij}(x_j))$ and weighting coefficients (b_{ij}) .

Referring to the sample matrix in Fig. 2 of my previous memorandum [6], the b_{ij} 's are the entries in this matrix, the a_i 's are the weighting coefficients listed to the left (or input end) of the matrix, while the weighting coefficients at the top (or output end) of the matrix are the $(\sum_i a_i b_{ij})$'s, one for each index j . The functions $(v_{ij}(x_j))$ were not derived in the memorandum.

The extension of this approach to more than one intermediate level of attributes is conceptually clear but has not been worked out specifically.

The required assumptions expressed in Eqs. (37) and (38) can be interpreted as follows. The effect of the output of an intermediate attribute on the worth function is linear and independent of the output level of any other intermediate attribute. The effect of the output of an outcome variable on the output of an intermediate attribute is independent of the output level of any other outcome variable. However, the effect of any outcome variable is not necessarily linear.

Explicit alternate means, within the context of space mission programs, of identifying whether any attribute satisfies the independence conditions need to be determined in order to provide some insight into the implementation feasibility of this approach.

References

1. Miller, J. R., "Assessing Alternative Transportation Systems, RM-5865-DOT, The RAND Corporation, Santa Monica, Calif. 1969.
2. Raiffa, Howard, "Preferences for Multi-Attributed Alternatives," RM-5868-DOT/RC, The RAND Corporation, Santa Monica, California, 1969.
3. Pollak, R. A., "Additive von Neumann-Morgenstern Utility Functions," *Econometrica*, Vol. 35, No. 3-4, July-Oct., 1967.
4. Keeney, R. L. "Quasi-Separable Utility Functions," *Naval Research Logistics Quarterly*, Vol. 15, No. 4, Dec., 1968.
5. Boyd, D. W., "A Methodology for Analyzing Decision Problems Involving Complex Preference Assessments," Doctoral Dissertation, Stanford University, Jan. 1970.
6. Olender, H. A., Memorandum to Jerry Deerwester dated July 29, 1970.

Further Development of the Mathematical Structure of the Multi-Level, Multi-Attributed Problem

The previous memorandum dealt with the development of the mathematical structure of the multi-level, multi-attributed problem. The mathematical development will be extended in this memorandum to make more explicit and interpretable some of the parameter of the mathematical model.

First let us consider the mathematical structure of Miller's worth scoring approach. We start by assuming an additive form for the worth function

$$w = \sum_{j \in J} f_j(x_j) \quad (1)$$

where J is the set of indices denoting the elements of the outcome variable vector \underline{x} .

We now define $\underline{x}_* \equiv (x_{1*}, x_{2*}, \dots) \equiv$ an outcome vector whose elements constitute a logical or feasible lower bound for each outcome variable element:

$\underline{x}^* \equiv (x_1^*, x_2^*, \dots) \equiv$ an outcome vector whose elements constitute a logical or feasible upper bound for each outcome variable element.

Letting

$$a_j = f_j(x_j^*) - f_j(x_{j*}) \quad , \quad (2)$$

we can write equation 1 as

$$w = \sum_j f_j(x_{j*}) + \sum_j a_j g_j(x_j) \quad (3)$$

Since we are only interested in w to an additive or multiplicative constant we can define

$$w' = \sum_j a_j g_j(x_j) \quad (4)$$

Note that each function $g_j(x_j)$ has a lower bound equal to zero and an upper bound equal to unity.

Miller assumes a hierarchial structure which divides the outcome variable elements into separate groups. This structure (for the case involving only one additional intermediate level of objectives) is indicated mathematically by defining

$$J \equiv J_1 \cup J_2 \cup \dots \cup J_N \quad (5)$$

such that

$$J_i \cap J_j \text{ is empty for } i \neq j \quad (6)$$

Thus,

$$w' = \sum_{i \in I} \sum_{j \in J_i} a_j g_j(x_j) \quad (7)$$

Through a series of substitutions involving the following definitions,

$$m_i \in J_i \quad (8)$$

$$b_{ij} \equiv \frac{a_j}{a_{m_i}} \quad (9)$$

$$b_i \equiv \sum_{j \in J_i} b_{ij} \quad (10)$$

$$c_i \equiv \sum_{J_i} a_j \quad (11)$$

$$n \in I \quad (12)$$

$$d_i \equiv \frac{c_i}{c_n} \quad (13)$$

$$d = \sum_{i \in I} d_i \quad (14)$$

we can rewrite equation 7 as

$$w' = c_n d \sum_{I} \frac{d_i}{d} \sum_{J_i} \frac{b_{ij}}{b_i} g_j(x_j) \quad (15)$$

(Note that the summation indexes, $j \in J_i$, and, $i \in I$, have been abbreviated to simply J_i and I .) Since we are not interested in the multiplicative constant, $c_n d$, we can define

$$w'' = \frac{w'}{c_n d} = \sum_I \frac{d_i}{d} \sum_{J_i} \frac{b_{ij}}{b_i} g_j(x_j) \quad (16)$$

The interpretation of the constants d_i , and b_{ij} is the matter of interest. First consider b_{ij} . We have

$$b_{ij} \equiv \frac{a_j}{a_{m_i}} \quad (17)$$

and from equation 4 we see that

$$a_j = \frac{\partial w''}{\partial g_j} \quad (18)$$

Thus,

$$b_{ij} = \frac{\partial w'' / \partial g_j}{\partial w'' / \partial g_{m_i}} = \frac{\partial g_{m_i}}{\partial g_j} \bigg|_{w'' = \text{const.}} \quad (19)$$

In Miller's hierarchical structure, this states that b_{ij} is the constant worth (or indifference) trade-off ratio between the achievement of the lowest level objective attributes (each of which is measured by an outcome variable element). These trade-offs are made only within a group of lowest level objective attributes supporting a common higher level objective attribute. It is also interesting to note that $a_j = \frac{\partial w}{\partial g_j}$, so that the validity of dropping additive and multiplicative constants is substantiated. The constant, b_i , is simply a normalization constant.

Next consider the constant d_i . In order to interpret this constant we need to provide some additional structure to equation 4. Rewrite equation 4 as

$$w' = \sum_i c_i \sum_{j_i} \frac{a_j}{c_i} g_j(x_j) \quad (20)$$

Let

$$p_i(\underline{x}) = \sum_{j_i} \frac{a_j}{c_i} g_j(x_j) \quad (21)$$

Thus,

$$w' = \sum_I c_i p_i(\underline{x}) \quad (22)$$

We can then see that

$$c_i = \frac{\partial w'}{\partial p_i} \quad (23)$$

Thus

$$d_i = \frac{c_i}{c_n} = \frac{\partial w' / \partial p_i}{\partial w' / \partial p_n} = \frac{\partial p_n}{\partial p_i} \quad \left| \quad w' = \text{const.} \right. \quad (24)$$

In Miller's hierarchical structure, this states that d_i is the constant worth trade-off ratio between the achievement of two higher level objectives. again, the constant d is simply a normalization constant. Note also that

$$p_i(\underline{x}_*) = \sum_{j_i} \frac{a_j}{c_i} g_i(x_{j_*}) = 0 \quad (25)$$

and

$$p_i(\underline{x}^*) = \sum_{j_i} \frac{a_j}{c_i} = 1 \quad (26)$$

Recapitulating, we see that Miller has essentially postulated an additive (independence assumptions) and composite (hierarchical structure) function for the worth function. Furthermore, Miller also assumes that the worth function is linear in all intermediate objective levels. These postulations

and assumptions, allow the task of assessing a worth function to be decomposed into

- 1) Assessing real valued functions (g_i 's) on real variables (x_j 's).
- 2) Assessing trade-offs or weighting constants between intermediate level objectives.

One of the implications of Miller's approach is indicated by the assumption that

$$J_i \cap J_j \text{ is empty for } i \neq j \quad (27)$$

This states that any given outcome variable element supports one and only one higher level objective. This restriction may not be acceptable in certain decision problems. By dropping this restriction we can still decompose the worth assessment problem in a similar manner as above, but at the cost of assessing a greater number of functions of real variables. The approach dropping this restriction will be subsequently referred to as the Extended Miller approach.

To develop the mathematical structure we start with the following postulated form for the worth function

$$w = \sum_I f_i(\underline{x}) \quad (28)$$

where I is the index set corresponding to the intermediate list of objectives as before.

Assume, by independence,

$$f_i(\underline{x}) = \sum_J f_{ij}(x_j) \quad (29)$$

This leads us to write

$$w = \sum_I \sum_J f_{ij}(x_j) \quad (30)$$

Equation 30 differs from equation 1 in that equation 30 assumes that any given intermediate objective may be supported by any number of the set of outcome variable elements, whereas equation 1 assumes that only some subset of the outcome variable elements support a given intermediate level objective and no other.

By a similar development as above we can arrive at a stage where we can write

$$w' = \sum_I \sum_{J_i} a_{ij} g_{ij}(x_j) \quad (31)$$

where the index sets are no longer mutually exclusive, and the subscript i or j only indicates that there may exist outcome variable elements which do not support a given objective. The functions, g_{ij} , vary in the range of interest from zero to unity as before.

Again, through a series of substitutions involving the following definitions,

$$m_i \in J_i \quad (32)$$

$$b_{ij} \equiv \frac{a_{ij}}{a_{im_i}} \quad (33)$$

$$b_i = \sum_{J_i} b_{ij} \quad (34)$$

$$c_i = \sum_{j \in J_i} a_{ij} \quad (35)$$

$$n \in I \quad (36)$$

$$d_i = \frac{c_i}{c_n} \quad (37)$$

$$d = \sum_I d_i \quad (38)$$

we can rewrite equation 31 as

$$w' = c_n d \sum_I \frac{d_i}{d} \sum_{j \in J_i} \frac{b_{ij}}{b_i} g_{ij}(x_j) \quad (39)$$

This equation has almost precisely the same form as equation 15, with the exception of the extra subscript on g_{ij} .

The interpretation of the constants d_i , and b_{ij} are again trade-off ratios.

$$b_{ij} = \left. \frac{\partial g_{im_i}}{\partial g_{ij}} \right|_{w' = \text{const.}} \quad (40)$$

$$d_i = \left. \frac{\partial p_n}{\partial p_i} \right|_{w' = \text{const.}} \quad (41)$$

If the index set J runs up to M and the index set I runs up to N , then the number of functions g_{ij} which must be assessed is between M and $M \times N$ depending on the degree of overlap of the index sets, J_i .

This is in contrast to the unextended Miller approach where only M worth functions, g_i , need to be assessed.

Finally, it should be noted that equations 19 and 40 have alternate equivalent interpretations. In particular we can write equation 19 as

$$b_{ij} = \left. \frac{\partial g_{mi}}{\partial g_i} \right|_{p_i = \text{const.}} \quad (42)$$

and equation 40 as

$$b_{ij} = \left. \frac{\partial g_{im_i}}{\partial g_{ij}} \right|_{p_i = \text{const.}} \quad (43)$$

These last two relationships show that constant worth trade-offs are equivalent to constant intermediate objective achievement trade-offs.

The equations developed in this memorandum should serve to identify the type of trade-off questions that the decision-maker must respond to in order to implement the approach. The specific procedures (sets of meaningful questions for the decision-maker and the rules for using the answers) should now be formulated in the context of space mission goals and objectives. These procedures should include procedures for identifying independence of objectives and outcome variable elements, assessing trade-off ratios, and assessing worth functions on real valued variables. The next memorandum will deal with these later topics.

Selection of Outcome Variable Elements for
the Limited Manned Space Flight Pilot Study

By

Henry A. Olender

The purpose of this memorandum is to present the results of an initial attempt to structure a set of outcome variable elements for the Mission Analysis Division pilot study of a limited manned space flight program, consistent with the approach outlined by Richard D. Smallwood in his memorandum of April 30. The intent is to provide a distinction between outcome variables, and value variables as suggested in that memorandum. This distinction should clarify some of the steps in the evaluation process and provide visibility of the decision analysis structure for the decision maker.

Prior to presenting these initial results, I believe the suggested approach would be clarified by defining several terms.

An outcome variable is a vector variable whose elements are qualitatively specified in specific technological terms. For example, a simple two-dimensional outcome variable relating to manned space flight programs is one whose first element relates to the number of man-hours of space flight per year, while the second element relates to the maximum gap in manned space flight. An outcome is a vector whose elements correspond to the elements of the outcome variable, but are in some sense quantitatively specified. The quantitative measure used to specify each element of an outcome is not necessarily a number; it may be an absolute or relative number, or a qualitative statement about the absolute or relative quantitative output. Outcomes are specified for each alternative, and for each uncertain state. Thus, if we are considering four alternatives, there is one outcome variable vector, four outcome vectors when no uncertainty prevails, or $4 \times n$ outcomes when n states of uncertainty exist.

A value structure variable is a vector variable of the same dimensions as the outcome variable, whose elements are specified as the views or values of some element of the environment toward the corresponding elements of the outcome variable (not the outcomes). To pursue the previous two-dimensional outcome variable example, the first element of the value structure variable would be a statement about the preference of some element of the environment for man-hours in space, while the second element would be a statement about their concern for gaps in manned space flights. Value structures are derived by assigning a quantitative measure to each element of the value structure variable for each element of the environment. One candidate for quantitative measures in this context might be pluses, zeroes, and minuses. Thus, if there are m elements of the environment, there will be m value structures.

Finally, an outcome value variable is a vector variable of the same dimensions as the outcome variable, whose elements are specified as the value of the corresponding outcome variable elements based on the quantitative measure of both the corresponding outcome elements and value structure elements. As before, outcome values are vectors derived by assigning quantitative measures to the elements of the outcome value variable. An outcome value is obtained for each alternative, and for each element of the environment. Thus, if there are four alternatives and m elements of the environment, there will be $4 \times m$ outcome values.

The distinctions outlined above between outcomes, value structures, and outcome values results in the following clarification of the steps in the evaluation process.

1. It allows the evaluation of outcome variable elements to be carried out independently^{*} of the alternative programs and their outcomes.

* Selection (as opposed to evaluation) of outcome variable elements is of course accomplished with the alternative programs in mind.

2. It allows the decision maker to see whether entries in a final evaluation matrix, such as Table I of the M.A.D. memorandum of March 20, 1970, represent failure or success on the part of an outcome to supply that outcome variable, or a negative or positive view on the part of the environment toward the particular outcome variable.

Thus, in this approach, the first step is to identify the elements of the outcome variables. A useful organizing framework, (at least for me) is to initially construct a list of goals to which space programs relate, and then for each goal list outcome variable elements which support the goal. There will, of course, be outcome variable elements which support more than one goal, and this degree of overlap provides useful information on the relative importance of these outcome variable elements. Categorizing outcome variables by goals they support is an attempt to identify and focus attention on the more significant outcome variable elements since, in the final analysis whether considered explicitly or implicitly, it is the degree of correlation between environment goals and outcomes that determines the outcome values.

The goals and outcome variable elements listed in this memorandum represent a first cut at this approach and may thus be not quite complete in some areas, or too extensive in other areas. It is expected that several iterations will be required to produce an outcome variable list that is both comprehensive enough and sufficiently limited to significant factors.

The goals that appear to be pertinent to this problem are the following:

1. Scientific Advancement
2. Technological Advancement
3. Achievement, per se
4. Defense Posture Improvement
5. Economic Benefits
6. National Prestige

7. Foreign Relations Improvement
8. Advancement of Long Range Space Programs
9. Political Posture Improvement

The following is a list of outcome variable elements categorized by the goals they support:

1. Scientific Advancement
 - a. Scientist-hours in space
 - b. Specific quantities of earth-oriented scientific data gathered.
 - c. Specific quantities of lunar-oriented scientific data gathered.
 - d. Specific quantities of solar system-oriented scientific data gathered.
2. Technological advancement
 - a. Man-hours in space
 - b. Advancement of earth orbital operational capability
 - c. Advancement of lunar station operational capability
 - d. Advancement of man's ability to perform new tasks in space.
 - e. Advancement of trans-orbital operational capability.
 - f. Advancement of earth-orbital rescue capability.
3. Achievement, per se
 - a. "Space firsts"
 - b. (Probably the same items listed under Technological Advancement).
4. Defense Posture Improvement
 - a. (Probably the same items listed under Technological Advancement with exception of 2c).
5. Economic Benefits
 - a. Estimated economic value of specific services made available in areas such as:
 1. forestry
 2. agriculture

3. weather
4. etc.
6. National Prestige
 - a. "Space firsts"
 - b. Man-hours in space
 - c. Scientist-hours in space
 - d. Number of scientists in space
 - e. Maximum gap in manned space flight
 - f. (Probably the same items listed under Technological Advancement)
7. Foreign Relations Improvement
 - a. Number of foreign nations cooperating or participating in program
 - b. Services rendered to foreign nations (see Economic Benefits)
 - c. Specific effects on competitive posture vis-a-vis the U.S.S.R.
 - d. Specific effects on national sovereignty of other nations.
 - e. Specific effects on our leadership posture.
8. Advancement of Long Range Space Programs
 - a. Estimated launch date for space station
 - b. Estimated launch date for space shuttle
 - c. Quantity of design and operational data gathered relating to space station
 - d. Quantity of design and operational data gathered relating to space shuttle
 - e. Estimated improvement of NASA's competitive position for funding.
9. Political Benefits
 - a. Effect of a program change
 - b. (see National Prestige)
 - c. (see Achievement)

At this point, the organizing framework of goals need not be retained. Redundancy in the outcome variable elements listed can now be eliminated to obtain the pertinent outcome variable vector. On the other hand, we may retain the information of the degree of redundancy obtained as bearing on the relative importance of the outcome variable elements.

AN APPROACH TO CORRELATING GROUP VALUES
TO OUTCOME ELEMENTS

by

Henry Olender

Previous memoranda have dealt with structuring the decision problem of the Limited Manned Space Flight Pilot Study in terms of outcome, value structure, and outcome value vector spaces. Alternatives are then defined on the outcome vector space; the elements of the environment are defined on the value structure space; and the interaction between the alternatives and the environment are defined on the outcome value space. If one is successful in meaningfully defining the outcome and value structure vector spaces to capture the essential factors bearing on the decision problem, then the decision problem structure can be effectively displayed in the outcome value space. This would bring us to the point where we would have provided a rational and visible structure to the problem and organized all relevant information available to us. Even if further development of the decision analysis methodology were not forthcoming, this structure and organization of information would be very valuable to a decision maker.

Two very important initial steps required in this approach are:

- (1) Identification of meaningful outcome variable elements, and
- (2) Modeling the value structure of the elements of the environment on those outcome variable elements.

The purpose of this memorandum is to discuss an approach to modeling the value structures of the elements of the environment. This approach can also be useful in identifying or selecting meaningful outcome variable elements.

The basic problem is this. Given a list of outcome variable elements and some specific group of people which constitute an element of the environment, we need to encode in some sort of quantitative statement the direction and intensity of the preference or concern of the group toward each outcome variable element. This will undoubtedly involve subjective evaluations by analysts, but at least we can enlist the aid of expert analysts who have studied the attitudes and values of various groups of people.

A serious difficulty is that the outcome variable elements will generally be defined at a level of high concreteness and oriented toward alternative space program outputs while the attributes of a group are defined at a level of high abstractness and oriented toward ideal goals. The result is a gap between the things we are trying to correlate due to the levels at which they are defined.

An approach that may be effective in bridging this gap is to construct several sets or lists of values, goals, or objectives (generically, we will call the elements of these sets attributes) which are defined at various levels of abstractness (or concreteness) intermediate between the values characterizing elements of the environment and the outcome variable elements. Each level would be progressively less abstract (or more concrete) and more oriented toward space program outputs (or less oriented toward ideals). We would then progressively attempt to correlate the items in a list at any given level to items in a list at the next highest level of abstractness. This correlation process becomes more feasible compared to the original correlation problem since we are now working at each stage with two lists of attributes which differ in level of abstractness and orientation by a relatively small degree.

The correlation process might be carried out by constructing a tree whose branches connect the items in a list of attributes at one level with those at the next level between which significant correlation exists. The degree of correlation may then be indicated by assigning correlation coefficients to each of the branches. By collapsing this tree (in a manner to be discussed later in this memorandum) from that level at which we can best construct a value model of the various elements of the environment to the outcome variable level, we will have for each element of the environment a set of weighting factors corresponding to the outcome variable elements. This set of weighting factors would constitute the value structure of that element of the environment.

Alternately, the process can be carried out in matrix form, where each matrix is a matrix of correlation coefficients between two lists of attributes defined at adjacent levels.

Assigning correlation coefficients to such a tree or series of matrices is perhaps the most difficult part of the problem and indeed we may not have developed a systematic methodology for doing this task effectively during the next iteration of the pilot study. However, it does appear feasible and useful to at least construct the tree showing the connections or branches linking the outcome variable elements and elements of the environment with respect to their preferences or concerns. Any subsequent subjective quantitative statements concerning the value structures can be checked for consistency with the connectedness structure of the tree. Thus, even without correlation coefficients, the connectedness structure could be useful to the decision analysts and the decision maker.

To clarify the approach outlined above an initial attempt was made to identify and describe the levels of attributes applicable to the pilot study, and construct candidate lists of attributes at each level. Also, the construction of an abbreviated correlation tree is attempted as an illustration.

My initial thoughts on the problem resulted in the identification of five levels of attributes which may be applicable to the pilot study. These five levels are identified and briefly described in Table 1. The selection of five levels is somewhat arbitrary at this point. It represents an attempt to provide enough levels so that any two adjacent levels are not too far removed from each other, and to keep the problem structure down to a manageable size. Obviously, one of these levels at or near the bottom of this list must correspond to the perceived outcomes of the various alternatives, while another level near the top of this list must be a convenient and meaningful level at which to model each of the elements of the environment.

Table I

Five Attribute Levels

1. Ideals - most abstract, motivating factors
2. Values - abstract, ideals oriented
3. Goals - less abstract, real world oriented
4. Space objectives - even less abstract, long range space program oriented
5. Outcome variables - specific, alternatives oriented

Table II is a list of items which constitute the first level of attributes. This list is an attempt to identify those human behavior motivating factors which are most widely accepted, and which all

individuals and groups share in various degrees. This first level is viewed as providing an organizing framework which is useful as a starting point, but probably not useful as a level at which to model the elements of the environment.

Table II

• List of Ideals

1. Freedom - least constraints on thoughts or actions
2. Equality - equality of opportunity
3. Power - control of destiny of others
4. Prestige - respect and esteem from others and oneself
5. Peace - absence of physical violence
6. Security - assurance of ability to maintain a desired state
7. Welfare - ability to obtain adequate food, shelter, and other comforts
8. Knowledge - satisfaction of awareness of facts and concepts
9. Technology - ability to do things
10. Pleasure - physical or intellectual
11. Love - emotional attachment based on intrinsic human attributes

Table III is a list of items at the "values" level. This level is meant to be the level at which the elements of the environment might be modeled, and in fact, is a list of values derived in the Hudson Report for this purpose. Several values listed in the Hudson Report such as "freedom," "equality," etc. have been left off this list since they are repeats of items from the previous list in Table II. Repeated items at

two different levels do not necessarily present any conceptual problem if this fact simply indicates that no useful distinguishing values can be identified at the lower level to resolve different aspects of an item at the higher level. At this point I do not feel that this is the case for the various elements of the environment we are considering. For example, it appears to me that there are different aspects of "freedom" and "equality" which distinguish how different elements of the environment view these ideals.

The list in Table III is constructed of single word descriptors of values which are meant simply to summarize the types of items that may make up the second level of attributes. Definition of each item in this list is required to clarify how elements of the environment may be modeled at this level. The basic purpose of these values is to resolve which aspects of the many aspects of each of the ideals in Table II best characterize the view of the various elements of the environment we are concerned with. For example, one group of people may view "expertise" as the primary means of increasing their prestige, while another group may view "intelligence" or "formal education" as their means of increasing prestige.

A review of the list of values in Table III will indicate that this list is perhaps too heavily loaded with values such as "conformity," "courtesy," etc. which will have very little correlation with the space program attributes. These items should probably be deleted from the list since we are only interested in modeling the elements of the environment to that degree of complexity which is sufficient for the decision problem at hand. Also there may be pertinent values which need to be added to this list.

Table III

List of Values

1. Expertise	20. Spontaneity
2. Planning	21. Creativity
3. Practical Accommodation	22. Community
4. Innovation	23. Generosity
5. Success	24. Concord
6. Social Stability	25. Patriotism
7. Continuity	26. Nationalism
8. Long-term Investment	27. Prudence
9. Intelligence	28. Conformity
10. Formal Education	29. Loyalty
11. Moderation	30. Competence
12. Individualism	31. Courtesy
13. Glory	32. Cooperation
14. Honor	33. Solidarity
15. Discrimination	34. Skill
16. Courage	35. Humility
17. Strength	36. Heart
18. Ideas	37. Self
19. Analysis	38. Immediacy

Table IV is a list of attributes at the "goals" level. This level is mean to be more oriented toward "real world" goals that interact with large scale technological programs. Most of these goals are national goals oriented as might be expected since they are most likely to interact

with large scale federally sponsored technological programs. However, several of the goals at the bottom of this list indicates that these goals are not and should not be limited strictly to national goals.

Table IV

List of Goals

1. U.S. achievement per se
2. Increasing U.S. prestige
3. Success in U.S.-U.S.S.R. competition
 - a. Economic
 - b. Military
 - c. Ideology
 - d. Science and technology
4. Increasing scientific knowledge
5. Advancing technological capability
6. Providing a "mobilization base" for long range continuance of technological capability
7. International cooperation
8. International trust
9. Economic assistance to other nations
10. Providing more equivalent to war
11. Direct economic benefits from technology
12. Economic benefits from stimulation of economy
13. Entertainment of public
14. Redirecting attention of society
15. Advancing personal careers
16. Advancing organizational interests
17. Maintaining funding for programs

Table V lists the attributes of the next level which are designated as "space objectives." These are more specific goals and are oriented toward space programs.

Table V

List of Space Objectives

1. Man on moon
2. Exploration of moon
3. Scientific experiments on moon
4. Man in space
5. Interplanetary astronomy
6. Scientific experiments in space
7. Technology experiments in space
8. Space applications (earth resources survey, weather, communications, navigation, etc.)
9. Operational capability in space
10. Space "firsts"

Finally, Table VI lists the outcome variable elements. These items are very specific and generally quantifiable. They supply the final link with the alternatives being considered, since each alternative is specified by assigning a quantitative measure or some qualitative statement about each of these outcome variable elements.

Table VI

List of Outcome Variable Elements

1. Man-hours on moon
2. Total hours on moon
3. Maximum length of stay on moon
4. Maximum number of men on moon simultaneously
5. Number of "rover" vehicles on moon
6. Number of different landing sites
7. Maximum radius of excursion from landing site
8. Maximum depth of core samples returned
9. Scientist hours on moon
10. Bits of scientific data returned (by category)
11. Number of remote scientific instruments on moon
12. Man-hours in space (earth orbit)
13. Total hours in space
14. Maximum mission duration
15. Maximum number of men in space simultaneously
16. Maximum gap in manned space flight
17. Number of EVA's
18. EVA man-hours
19. Scientist-hours in space
20. Bits of data returned
 - a. Scientific (further subdivided by category)
 - b. Technological (" " " ")
 - c. Application (" " " ")
21. Manhours expended on experiments (by type and category, as above)
22. Number of different experiments performed (by type)
23. Area of earth surveyed

Table VI (Continued)

24. Frequency of earth survey coverage
25. Resolution of earth survey data
26. Advance in launch date of space station
27. Advance in launch date of space shuttle
28. Number of rendezvous and dockings
29. Number of crew transfers between spacecrafts
30. Number of interim space station reactivations
31. Man-hours of artificial gravity operation
32. Degree of change in space program
33. Expected number of television viewer-hours of space activities
34. Bits of data returned on foreign experiments (by category)
35. Man-hours expended on foreign experiments (by category)
36. Number of hours audio/visual educational activities from space to earth
37. Potential for violation of national sovereignty of foreign nations through communications from space
38. Number of space "firsts"

An examination of this list will show that it is probably too long since it contains outcome variable elements which are either highly redundant in terms of the type of output they measure, or do not distinguish between the alternatives. The main purpose of this initial attempt at constructing such a list is to clarify what types of outputs are or should be viewed as outcome variable elements, and to be as comprehensive as possible. Later iterations should weed out some items and point up areas of oversight where new items should be added.

It should be noted that outcome variable elements need not all be required to support at least one of the space objectives listed in Table V. It may be appropriate to list items which support the higher level goals in Table IV which cannot be meaningfully expanded in terms of space objectives. For example, the degree of change in an existing space program may be an outcome variable element which does not interact with any of the space objectives but may indeed affect the goal of advancing one's personal career, or an organization interest.

It should also be noted that the outcomes of interest to a decision maker may most effectively be defined at some higher level than the specific type of outputs defined at the outcome variable level. Thus, the labels assigned to the various levels of attributes are not significant. The significant feature of the correlation tree is that it provides visibility to the structure linking abstract ideal goals or values with specific technological outputs of alternatives.

As a specific example to illustrate the output of such an approach, consider a case where previous analysis indicates that Group A is primarily concerned with advancing U.S. technology and success in U.S.-U.S.S.R. competition, while Group B is primarily concerned with increasing scientific knowledge, and advancing U.S. technology. Further, assume that the space objectives consist only of exploration of the moon, man in space, and operational capability in space, and that the pertinent outcome variable elements required to encode the outputs of our alternatives are those listed in Fig. 1. Also assume that the weighting factors on all branches emanating from a given node in the tree shown in Fig. 1 are equal. After normalizing so that the sum of all weighting factors associated with the branches connected to a single node equals one, the fractions shown in Fig. 1 near each branch result.

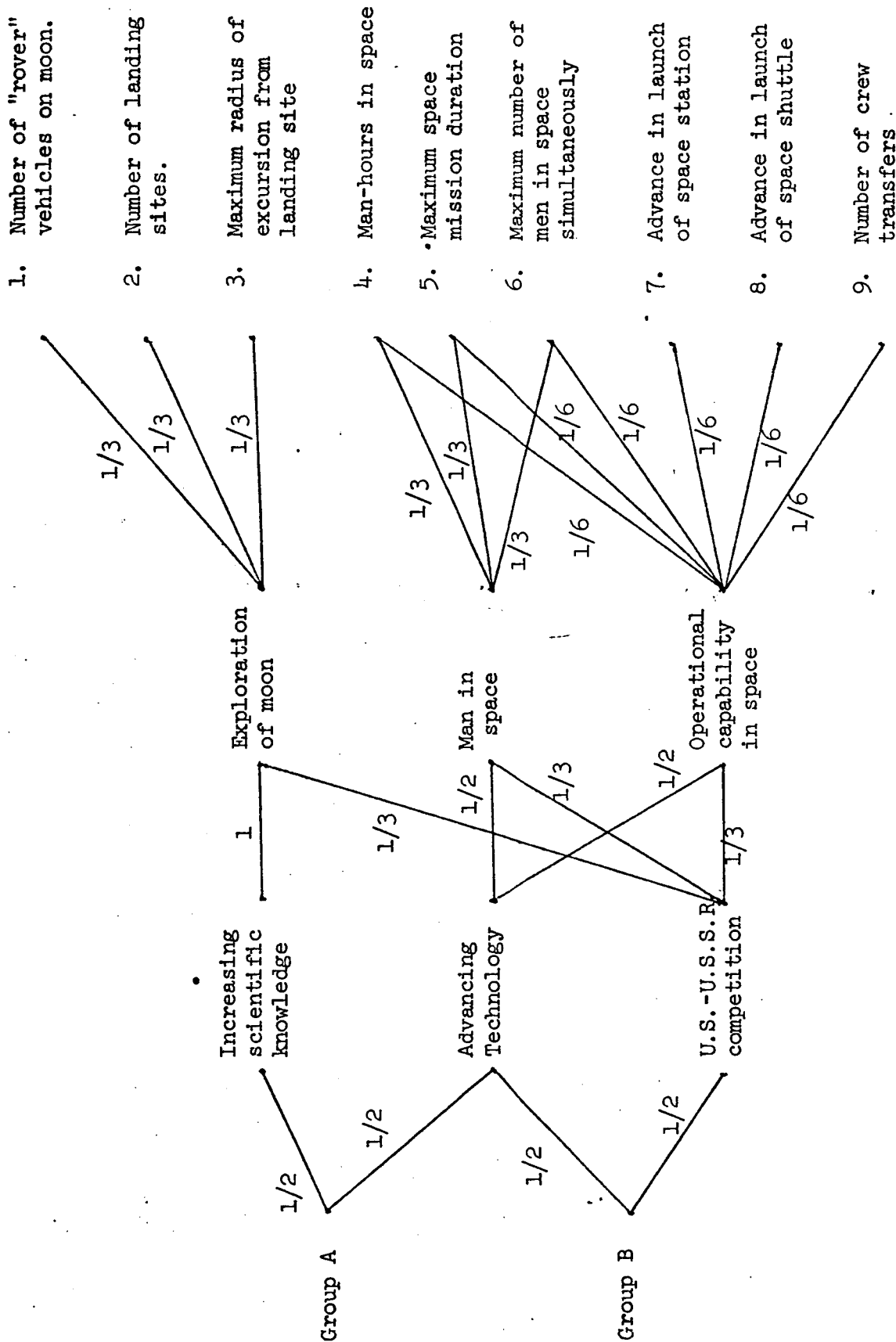


Fig. 1. Sample Correlation Tree.

To obtain the value structure vector corresponding to each element of the environment, we need to collapse or roll the tree forward from each element of the environment (deleting all other elements of the environment) to the outcome variable level. We do this by successively eliminating each level of nodes (i.e. either the first single node level corresponding to one particular element of the environment, or one of the defined intermediate level of attributes) in the following manner.

Replace the weighting factor associated with each branch emanating from each node at the level next below the level to be eliminated with the product of the old weighting factor and the sum of all weighting factors associated with branches entering the node. Finally, when the point is reached where only the outcome variable level and the next higher level of attributes remain, simply sum the weighting factors associated with all branches entering each outcome variable node (this is equivalent to considering a single branch emanating from each outcome variable node with unity weighting factor, and applying the above procedure). The result is a set of weighting factors, each corresponding to an outcome variable element. This set of weighting factors can be considered the value structure vector associated with a given element of the environment since they measure the relative concern or preferences of that group toward the outcome variable elements.

The results of this sample exercise using the information contained in Fig. 1 is tabulated in Table VII in the form of the value structure matrix. Group A is indicated to be most concerned with outcome variable elements, 4, 5 and 6, since these support both man in space and operational capability in space which both, in turn, support each of the basic concerns of Group A toward technology and U.S.-U.S.S.R. competition. On

Table VII

Sample Value Structure Matrix

Outcome Variables

Element of the Environment	1	2	3	4	5	6	7	8	9
Group A	4/72	4/72	4/72	15/72	15/72	15/72	5/72	5/72	5/72
Group B	12/72	12/72	12/72	9/72	9/72	9/72	3/72	3/72	3/72

the other hand, Group B is most concerned with outcome variable elements 1, 2, and 3 which are the only outcome variable elements which support their concern for advancing scientific knowledge. However, the outcome variable elements of next most concern are also 4, 5, and 6 since these most support the other concern of Group B toward advancing technology.